PRODUCTION AND OPERATIONS MANAGEMENT
Vol. 5, No. 3, Fall 1996
Printed in U.S.A.

TECHNICAL NOTE

A PROFIT-MAXIMIZATION MODEL FOR A RETAILER'S STOCKING DECISIONS ON PRODUCTS SUBJECT TO SUDDEN OBSOLESCENCE*

PRAFULLA JOGLEKAR AND PATRICK LEE
Department of Management, La Salle University, Philadelphia, Pennsylvania 19141, *USA*
Department of Management, Fairfield University, Fairfield, Connecticut 06430, *USA*

A product has been formally defined as being subject to sudden obsolescence if its lifetime is negative exponentially distributed. Using an *approximate* model, Masters suggested that the traditional method of incorporating obsolescence cost as a component of inventory holding costs in the economic order quantity (EOQ) model was appropriate for products subject to sudden obsolescence, provided that the obsolescence cost component was computed properly. Unfortunately, current practice of the EOQ model seriously underestimates the costs of sudden obsolescence. An *exact* model demonstrating that Masters' model also underestimated true lifetime costs and overestimated the optimal order quantity has been presented. Neither of these models addressed quantity discounts. Furthermore, with their cost-minimization focus, these models fail to identify situations when minimized costs exceed expected revenues. We extend Joglekar and Lee's model to focus on maximizing profits, rather than minimizing costs. This model answers such questions as whether to stock the product at all, whether to accept a quantity discount offer, and what order quantity to use. Numerical examples and sensitivity analyses suggest that Masters' model provides a significant improvement over the traditional model in moving toward true optimality. However, they also illustrate situations where both the traditional and the Masters' model accept a quantity discount that deserves to be rejected and stock a product that should not be stocked. In such situations, it seems important that a retailer uses the profit-maximization model presented here.
(INVENTORY MODEL; SUDDEN OBSOLESCENCE; PROFIT-MAXIMIZATION MODEL; QUANTITY DISCOUNT)

Introduction

When a style good has a stochastically distributed demand during a single, relatively short selling period, optimal stocking decisions can be based on the *newsboy model* (Silver and Peterson 1985). However, when the demand pattern could conceivably continue well into the future, and inventory holding costs are not negligible, the traditional approach is to incorporate obsolescence as a component of holding costs in the economic order quantity (EOQ) model (Naddor 1966; Silver and Peterson 1985). Masters (1991) identified several other models (e.g., Barbosa and Friedman 1979; Brown 1982; Hill,

* Received May 1994; revisions received February 1995, January 1996, and June 1996; accepted July 1996.

1059-1478/96/0503/288$1.25

Girard, and Mabert 1989; Naddor 1966) that dealt with gradual rather than sudden obsolescence. Masters defined *sudden obsolescence* as a situation when a product's lifetime is negative exponentially distributed, and, consequently, the probability of obsolescence is constant at any time. Using an *approximate* model, Masters concluded that for products subject to sudden obsolescence, the use of the EOQ model was appropriate, provided that the obsolescence component was computed properly.

Unfortunately, current industry practice of estimating inventory carrying costs in general, and obsolescence costs in particular, is seriously deficient. Lambert and Stock (1993) report that, in many companies, inventory carrying costs have never been computed. Many managers simply use textbook percentages or industry averages. Lambert and Stock critically reviewed 13 different inventory carrying cost estimates that ranged from 12 to 35%, with capital costs as the largest single component and "inventory risk costs" (including obsolescence, damage, and shrinkage) as the smallest component. They provide several examples to show that accounting practices in most companies do not facilitate a good estimation of obsolescence costs.

Textbooks in EOQ theory often suggest obsolescence costs in the range of 1–3% per year of inventory value (Starr 1989; Heizer and Render 1996). However, there is little reliable empirical or theoretical support for that suggestion. After an extensive literature search, we came across only one empirical study (Lambert 1975) of inventory carrying costs. In the six U.S. firms studied, the obsolescence costs were on the order of 1% of the inventory value. Lambert's method was to express all inventory written off during a year as a percent of the average inventory, without differentiating among product classes. Yet, Austin (1977) found that only 634 of the possible 106,000 inventory classes at the Air Force Logistics Command contained obsolete items, with obsolescence percentages ranging from 0 to 100%, but most in the 1–2% category.

Lambert's estimates may be particularly inappropriate in situations of sudden obsolescence. First, his data are from the packaged goods industry where sudden obsolescence is far less likely. Second, his data are from 1970s, when the rate of social and technical change was relatively slow and steady. Product life cycles were several years, if not decades, long. Obsolescence set in gradually as technology evolved or consumer preference changed. When new products were introduced, rarely did older products become completely worthless overnight requiring an actual write-off. Often, they could still be sold in some worldwide market.

With rapidly evolving technology, abrupt changes in global political situations, and instantaneous dissemination of information in the worldwide market, product life cycles have decreased dramatically, and a number of products are at risk of becoming obsolete overnight. Swiss watches, world maps, breast implants, and Milli Vanilli records are just a few examples of this phenomenon. For retailers of such products, obsolescence costs must be estimated on a sounder theoretical and empirical basis and differentiated by product class.

In contrast to Lambert's estimate, Masters' (1991) obsolescence cost formula is based on a sound, albeit approximate, theoretical model. It shows that for a product with zero salvage, the proper obsolescence cost is the reciprocal of the product's expected life. Thus, an obsolescence component on the order of 1% was appropriate only if the product had an expected life of 100 years! Masters warned that in cases of short-life products, failure to use the proper formula could lead to costs that were 5–40% higher than the optimal costs.

Although it represents a significant improvement over the traditional model, Masters' (1991) model is approximate. Using an *exact* formulation, Joglekar and Lee (1993) have shown that Masters' model underestimates the true lifetime costs of a stocking policy while overestimating the optimal order quantity, and that the associated errors are substantial when the expected number of inventory cycles in a product's lifetime is small.

Thus, for short-life products, such as pop music or video games, using an *exact* model is important.

Manufacturers of products subject to sudden obsolescence often offer substantial quantity discounts so as to entice retailers into sharing the risk of obsolescence. Conner (1986) observes that one reason companies end up with excessive obsolete inventory is that they cannot resist significant price breaks. However, neither Masters nor Joglekar and Lee have evaluated their models in situations of quantity discounts. Masters' model may lead to suboptimal decisions in these situations insofar as acceptance of a quantity discount reduces the expected number of inventory cycles during a product's lifetime.

Finally, like the EOQ and Masters', Joglekar and Lee's model (1993) uses a cost-minimization approach. These models can prescribe an order quantity that minimizes total inventory costs, but cannot warn against stocking a product at all when the minimized costs exceed the expected revenues. As Silver and Peterson (1985) suggest, one must look at the profit in deciding whether to stock a product at all. Therefore, we present an *exact* model that focuses on maximizing profits rather than on minimizing costs. The results of our model are compared with those of the traditional model (i.e., the EOQ model with obsolescence costs in the range of 1–3%), as well as those of Masters' (1991) model. Masters' model represents a significant improvement over the traditional model. However, our model promises additional improvement in profits, particularly in situations of quantity discounts for products with short lifetimes and small salvage values.

Notation

B = break point for the quantity discount,

C_d = unit cost of the item when $Q \geq B$. We assume $C_d \leq C_p$,

C_o = ordering costs per order,

C_p = unit cost of the item when $Q < B$,

C_Q = unit cost of the item when the order quantity is Q,

D = the constant demand in units per year until t,

H_n = annual holding costs (other than obsolescence) per dollar of inventory,

H_o = annual obsolescence cost per dollar of inventory,

k = probability, at the beginning of an order cycle of Q/D years, that the product does not become obsolete during the order cycle,

L = expected life of the product in years,

P_L = expected lifetime profit as estimated by the traditional model,

Q = order quantity per order prior to obsolescence at time t,

Q_d = order quantity per order at the discounted unit cost of C_d,

Q_p = order quantity per order at the regular unit cost of C_p,

S_p = selling price per unit prior to obsolescence,

S_o = selling price (salvage value) per unit after obsolescence, $S_o \leq C_d \leq C_p$,

t = the time at which the product becomes obsolete,

$\text{TRC}(Q)$ = annual total relevant costs of ordering, holding, obsolescence, as well as the purchase cost of D units (with Q as the order quantity),

$E[P_c(Q)]$ = profit during one inventory cycle, as expected at the beginning of the cycle,

$E[P_L(Q)]$ = lifetime profit, as expected at the beginning of an order cycle of Q/D years.

Not all of the following models use each of these variables. The traditional and Masters' models do not use $E[P_c(Q)]$ or $E[P_L(Q)]$ and our model does not use H_o or $\text{TRC}(Q)$.

The Traditional and Masters' Models

The traditional model is the EOQ model with H_o in the range of 1–3%. The smaller the value of H_o, the worse the traditional model would look compared to Masters' and

Joglekar and Lee's (1993). Hence, in order to give the full benefit of doubt to the traditional model, we assume in our numerical examples that $H_o = 3\%$. Assuming that H_o adequately captures the impact of such factors as L or S_o, the traditional model seeks to obtain the optimal Q by minimizing the total relevant cost (TRC):

$$\mathrm{TRC}(Q) = DC_Q + DC_o/Q + QC_Q(H_n + H_o)/2. \tag{1}$$

For a comparison with our model, we assume that at the beginning of an inventory cycle the traditional model estimates the expected lifetime profit by the formula

$$P_L^* = DLS_p - L[\mathrm{TRC}^*]. \tag{2}$$

where TRC* is given by (1) using Q_d^* or B or Q_p^* as appropriate.

Masters (1991) endorsed the use of the EOQ model, provided H_o was estimated as

$$H_o = (C_Q - S_o)/(C_Q L). \tag{3}$$

Thus, we assume that Masters model's decisions and consequences are given by the above logic, once (3) is substituted in (1) and (2).

A Profit-Maximizing Model

We extend Joglekar and Lee's (1993) exact-cost methodology to the consideration of expected lifetime profits resulting from a time-invariant order of Q units. When an inventory cycle of Q units begins, the ordering cost, C_o, and the purchase price of Q units, QC_Q, are incurred with certainty, representing an outlay of

$$C_o + QC_Q. \tag{4}$$

Product revenues and holding costs depend upon whether and when the product becomes obsolete during an inventory cycle. If the product does not become obsolete during an order cycle, that is, if $t \geq (Q/D)$, all Q units sell at price S_p for a cycle revenue of QS_p. The corresponding holding costs are $(Q/2)(Q/D)(C_Q H_n)$. These revenues and costs are obtained with the probability that $t \geq (Q/D)$. The corresponding expected profit contribution is

$$\int_{Q/D}^{\infty} [(QS_p) - (Q/2)(Q/D)(C_Q H_n)][(1/L)e^{-(t/L)}]\mathrm{d}t. \tag{5}$$

Using Joglekar and Lee's (1993) (A.2), this expression can be simplified to

$$[QS_p - Q^2 C_Q H_n/(2D)]k. \tag{6}$$

If obsolescence occurs at time t ($t \leq Q/D$), only tD units sell at S_p, and $(Q - tD)$ units sell at S_o. Thus, the revenue during the inventory cycle is $[tDS_p + (Q - tD)S_o]$, and holding costs are $[Q - (tD/2)](C_Q H_n t)$. The corresponding expected profit contribution is

$$\int_0^{Q/D} \{[tDS_p + (Q - tD)S_o] - [Q - (tD/2)](C_Q H_n t)\}[(1/L)e^{-(t/L)}]\mathrm{d}t. \tag{7}$$

Using Joglekar and Lee's (1993) (A.1)–(A.4), expression (7) can be simplified to

$$(S_p - S_o + C_Q H_n L)DL[1 - k] + Q[S_o - C_Q H_n L] - [QS_p - Q^2 C_Q H_n/(2D)]k. \tag{8}$$

Then, $E[P_c(Q)]$, the profit during one inventory cycle of Q/D years, as expected at the beginning of that cycle, is given by the sum of expressions (4), (6), and (8). Thus,

$$E[P_c(Q)] = (S_p - S_o + C_Q H_n L)DL[1 - k] + Q[S_o - C_Q H_n L] - C_o - QC_Q. \tag{9}$$

Whether or not product obsolescence occurs during a given inventory cycle, $E[P_c(Q)]$ gives the sum of all revenues and costs accruing during that cycle, weighted by their respective probabilities of occurrence, as estimated at the beginning of the cycle.

Given a constant obsolescence rate and a time-invariant order quantity, the product's lifetime profit as expected at the beginning of an order cycle is identical to that at the beginning of the next order cycle. Hence, if k is the probability of the product surviving past one inventory cycle, we have

$$E[P_L(Q)] = E[P_c(Q)] + kE[P_L(Q)]. \tag{10}$$

This can be simplified to

$$E[P_L(Q)] = DL(S_p - S_o + C_Q H_n L) + [(S_o - C_Q - C_Q H_n L)Q - C_o]/[1 - k]. \tag{11}$$

A careful comparison of our (11) to Joglekar and Lee's (1993) (11) reveals that the two models are consistent. Indeed, when we recognize that Joglekar and Lee's C_s is equivalent to our $C_p - S_o$, it can be shown that our $E[P_L(Q)] = DLS_p - C_L$, where C_L stands for Joglekar and Lee's expected lifetime costs.

There is no closed form solution for the profit-maximizing Q of (11). However, for specific parameter values, the software package GINO (Liebman, Lasdon, Schrage, and Warren 1986) solves the problem within seconds.

In a situation of quantity discount, the first step is to obtain Q_d^* by maximizing (11) when $C_Q = C_d$. If $Q_d^* \geq B$, ordering Q_d^* and taking the discount is better than not taking the discount. However, Q_d^* will be actually implemented only if $E[P_L(Q_d^*)] > 0$.

If $Q_d^* < B$, the discount is worth taking only if $E[P_L(B)] > E[P_L(Q_p^*)]$ and $E[P_L(B)] > 0$.

Numerical Examples

Table 1 presents two scenarios of quantity discounts for short-life products subject to sudden obsolescence. The demand, ordering cost, nonobsolescence holding cost, unit cost, and selling price are assumed to be the same in both scenarios. Each scenario's distinct assumptions are listed on top. Finally, Table 1 lists the decisions and consequences of each of the three models under each scenario.

Scenario 1 considers an expected life of 1 year, a low salvage value, and a modest quantity discount. Although their Q_d^*'s are smaller than B (=5,000 units), both the traditional and the Masters' models find it more economical to take the discount by ordering B, since their TRC(B's) are less than TRC(Q_p^*)'s. However, according to the profit-maximization model, the best thing to do is to reject the discount, and order only 1,936 units. As the bottom two lines of Table 1 indicate, in scenario 1, the traditional model overestimates the profit from its optimal policy ($19,355) by 110% (compared to the true profit of $9,221), whereas Masters' model overestimates it by 16%. The profit-maximization model's optimal order quantity results in an increase of $102 (or 1.1%) over the true profit resulting from either of the other models.

Scenario 2 considers an expected life of 3 months, zero salvage value, and a deep (40%) quantity discount. Here, the best decision is not to stock the product at all. Yet, cost-based models, such as the traditional model or Masters' model, would not normally consider the decision of stocking or not stocking. Even if these models were extended to include that decision, in this example, they would each estimate a positive profit at their respective optimal quantities. The traditional model's Q_d^* of 5,384 units would automatically qualify it for the discounted unit cost of $3, and its estimate of the lifetime *profit* of that decision would be $9,071. In actuality, that decision would result in a *loss* of $2,442. Masters' model would recommend an order quantity of 5,000 units to take advantage of the deep discount, with an estimated corresponding *profit* of $1,625. In reality, that decision would result in a *loss* of $1,497.

TABLE 1

A Comparison of Each Model's Decisions and Consequences Under Two Scenarios

Model's Decisions and Consequences/Model:	Scenario 1 (L = 1 year, S_o = \$1/unit, and C'_d = \$4.60/unit) Traditional	Masters'	Profit-Max.	Scenario 2 (L = 0.25 year, S_o = \$0/unit, and C'_d = \$3.00/unit) Traditional	Masters'	Profit-Max.
Q_p^* (units/order)	4,170	2,000	1,936	4,170	976	916
Q_d^* (units/order)	4,348	2,104	2,032	5,384	1,260	1,162
$Q_d^* \geq B$	No	No	No	Yes	No	No
TRC (Q_d^*) (\$/yr)	NA	NA	—	33,715	NA	—
TRC (B) (\$/yr)	50,645	59,300	—	NA	63,500	—
TRC (Q_p^*) (\$/yr)	54,796	60,000	—	54,796	70,494	—
TRC (B) < TRC (Q_p^*)?	Yes	Yes	NA	NA	Yes	—
$E[(P_L(Q_d^*)]$ (\$)	—	—	NA	—	—	NA
$E[(P_L(B)]$ (\$)	—	—	9,221	—	—	−1,497
$E[(P_L(Q_p^*)]$ (\$)	—	—	9,323	—	—	−811
$E[(P_L(B)] > E[(P_L(Q_p^*)]$?	—	—	No	—	—	No
Accept discount?	Yes	Yes	No	Yes	Yes	No
Optimal Q^* (units/order)	5,000	5,000	1,936	5,384	5,000	916
$E[P_L(Q^*)] > 0$?	—	—	Yes	—	—	No
Stock the product?	NA	NA	Yes	NA	NA	No
Model's estimate of lifetime profit (\$)	19,355	10,700	9,323	9,071	1,625	0
True lifetime profit at model's order quantity (\$)	9,221	9,221	9,323	−2,442	−1,497	0

Assumptions common to both scenarios: D = 10,000 units/yr, C_o = \$1,000/order, H_n = 0.2/\$/yr, S_p = \$7/unit, C_p = \$5/unit, B = 5,000 units/order, H_o = \$0.03/\$/yr in the traditional model, and $H_o = (C_Q - S_o)/(C_Q L)$ in Masters' Model.

In short, these numerical examples demonstrate the value of our profit-maximizing model. To assess model sensitivities, in each scenario, keeping all other assumptions constant, we (Lee and Joglekar, 1996) plotted each model's optimal order quantity and the resulting true profits as functions of S_o and L. Together these diagrams showed that Masters' model represents a significant improvement over the traditional model. However, for very small values of S_o and L, Masters' model may also suboptimize compared to our model, which yields the best profits under all circumstances.

Conclusion

Current EOQ practice seriously underestimates the cost of sudden obsolescence. Masters (1991) made an important contribution by proposing a better method of estimating this cost. For products with reasonably long expected lives and no quantity discounts, the EOQ model would serve well, provided obsolescence costs are estimated by Masters' formula. However, Masters' (1991) model is an *approximate* one, focusing only on cost minimization. We have presented an *exact* model that focuses on profit maximization in the context of such decisions as whether to stock a product at all, whether to take a quantity discount, and how many units to order. Through numerical examples and sensitivity analyses, we show that our model is particularly important when quantity discounts are available, when a product's expected lifetime is only several months long, and when the salvage value is negligible.

We have presented a simple extension of Joglekar and Lee's (1993) model. Several directions for further research can be noted. First, if our assumption that sudden obsolescence is characterized by a negative exponential lifetime is not valid, a time-invariant order quantity would not be optimal and our profit-maximization approach must be integrated with a dynamic programming model (e.g., Brown, Lu, and Wolfson 1964; and Nahmias 1974). Second, for fashion goods, our assumption of a deterministic demand is questionable. When demand is stochastic but the mean demand is stationary, it would be interesting to integrate our model with Hadley and Whitin's (1963) lot size, reorder point (Q, r) models. However, if the mean demand is not stationary, once again, a dynamic programming model would be necessary. Third, the selling price of fashion goods often depends on the observed demand during prior weeks. A model accommodating this demand-dependent price would represent another important direction for an extension of our model. Nose, Ishii, and Nishida's (1984) work may be relevant for such an extension. Fourth, style goods are often exchanged among retailers for other style goods, depending on who is selling more of what, indicating the need for a multiproduct model. Finally, an obsolete product today often acquires a collectors' premium tomorrow. To the extent that such a change in the product's valuation and its timing is predictable, it can influence one's order quantity decisions. We believe that in all these situations, focusing on profit maximization rather than cost minimization is the right direction for future research.

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