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NOTES

COMMENTS ON "A QUANTITY DISCOUNT PRICING MODEL TO INCREASE VENDOR PROFITS"*

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Monahan (1984) adapted the quantity discount model of inventory theory to the problem of determining an optimal quantity discount schedule from a vendor's point of view, and opened up an important direction of research. However, his one-item, one-customer, one-vendor model is based on several implicit assumptions that must be judged unreasonable. Monahan must account for the vendor's inventory carrying charges and redefine his variable S_2. It is shown that a rational vendor's manufacturing frequency would not be identical to the buyer's ordering frequency if the vendor's manufacturing setup costs are substantially larger than the buyer's ordering costs. A numerical example presented in this note also questions the practical usefulness of Monahan's model even after its theoretical inaccuracies are corrected. Monahan's model may explain discounts that are a fraction of 1% of the price of an item, but it fails to explain commonly observed magnitudes of quantity discounts, such as 10% of the unit price.
(INVENTORY/PRODUCTION—POLICIES, PRICING; MARKETING—PRICING; INVENTORY/PRODUCTION—DETERMINISTIC MODELS)

In a recent paper, Monahan (1984) adapted the well-known quantity discount model of inventory control to help a vendor determine an optimal discount pricing schedule. Monahan has commendably pointed to an important direction of research that management scientists seem to have overlooked. However, his model seems to have serious shortcomings both in its assumptions and its practical usefulness, as outlined below:

1. Unreasonable Implicit Assumptions

Although Monahan warned amply about the sizable risks in using his one-customer model in a multiple-customer situation, there are sizable risks in using his model in a one-customer situation as well, if the situation does not meet his implied assumptions, namely:

(A) the vendor's frequency of order processing and manufacturing set-ups is the same as the buyer's ordering frequency, and

(B) the vendor's inventory and inventory carrying costs are unaffected by the buyer's order frequency.

Now a portion of assumption (A) above seems reasonable. One would expect the vendor's *order processing* (but not manufacturing) frequency to be the same as the buyer's ordering frequency (although, an order may be a multi-item order rather than the single-item order assumed). However, it seems highly unlikely that the vendor's manufacturing set-up frequency will be the same as the buyer's ordering frequency unless:

(i) either (ia) the buyer is willing to accept a long lead time in getting his order fulfilled or

(ib) the buyer's order frequency is periodic and known to the vendor who therefore schedules his production in anticipation of the order,

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(ii) the vendor has substantial unused capacity (exclusively for the production of the item under consideration) that is simply waiting for the buyer's order (or anticipated order) to start the production of this item, without disturbing the production schedules of any other items the vendor may be manufacturing, and

(iii) the vendor's inventory carrying charges are so high and his manufacturing set-up costs are so low that the use of a production lot size greater than the buyer's order size is economically inferior to the policy of producing to order.

Insofar as some of the above conditions may not be true, and their simultaneous incidence may be unlikely still, assumption (A) above seems unrealistic.

Whether these conditions are realistic or not, Monahan certainly did not want to imply that the vendor's manufacturing set-up costs are low as suggested in (iii) above. In discussing the significance of his results, Monahan says:

> Somewhat surprisingly, for situations in which the two parties have similar costs ($S_2 \equiv S_1$) the model can entail order size revisions of more than 40 percent. Clearly, maximum "action" occurs when S_2 is substantially larger than S_1. This is most likely to occur in circumstances where the vendor produces the item internally, and hence must include manufacturing set-up costs, in his determination of the size of S_2. (p. 723)

In §6 below, I show that if manufacturing set-up costs are high, a vendor would be better off deviating from the policy of matching his production frequency to the buyer's order frequency. Thus, assumption (A) seems to be not only unrealistic but also irrational from a vendor's point of view, if Monahan's "maximum action" circumstances were to prevail.

In the interest of theoretical development, let us grant for the moment that assumption (A) is valid, i.e., the vendor produces only to the buyer's order. Even then, one cannot subscribe to Monahan's implied assumption (B) that the vendor's inventory carrying costs are unaffected by the buyer's order frequency.

2. Accounting for the Vendor's Inventory and Corresponding Inventory Carrying Costs

Let us define R_2 as the vendor's annual production capacity for the item under consideration. Then, every time the vendor produces Q units, he would spend (Q/R_2) years with an average inventory of $(Q/2)$ units. Using the symbols H_2, M_2, P_1, D_1, and N_1, as defined by Monahan, one can show that the vendor's inventory carrying cost during these production days in an order cycle would be

$$(1/2)Q(1 - M_2)P_1H_2(Q/R_2). \tag{1}$$

Clearly, these costs are a function of Q and would be different from different order sizes. If the buyer's order quantity is Q_1, and there are D_1/Q_1 order-cycles per year, the vendor's *annual* inventory carrying costs will be

$$(1/2)Q_1(1 - M_2)P_1H_2(D_1/R_2). \tag{2}$$

In view of these costs, Monahan's equation (8) for the vendor's annual profits under the no discount policy must be corrected to be

$$YNP_2 = D_1M_2P_1 - N_1S_2 - (1/2)Q_1(1 - M_2)P_1H_2D_1/R_2. \tag{3}$$

Similarly, his equation (9) for vendor's profits under the discount policy must be corrected to be

$$\mathrm{YNP}_2^1 = D_1M_2P_1 - D_1d_K - (N_1/K)S_2 - (1/2)(Q_1K)(1 - M_2)P_1H_2D_1/R_2. \tag{4}$$

Consequently, the corrected expression for the optimal value of K turns out to be

$$K = \sqrt{\{1 + (S_2/S_1)\}/\{1 + (H_2/H_1)(1 - M_2)(D_1/R_2)\}}. \tag{5}$$

Also the expression for d_K^1, the maximum discount the vendor can afford, becomes

$$d_K^1 = \{(S_2/Q_1K) - (1/2)Q_1(1 - M_2)P_1H_2/R_2\}(K - 1). \quad (6)$$

3. The Need To Redefine S_2

Of course, the corrections outlined in §2 would be unnecessary if Monahan had simply assumed that the vendor's production frequency was independent of the buyer's order frequency.[1] In such a case, the restrictive conditions for the fulfillment of assumption (A) discussed in §1 need not hold. It may also be reasonable to assume that the vendor's average inventory and his corresponding inventory carrying charges are not significantly influenced by the buyer's order frequency.[2] Then S_2 would be defined simply as the vendor's order processing costs (not including his manufacturing setup costs), and Monahan's model would be technically correct.

However, in this case, on the basis of my earlier quotation from Monahan, S_2 may be assumed to be of the same order of magnitude as S_1. Although Monahan suggests that even this situation would be a significant one in that it would "entail order size revisions of more than 40 percent," I believe that the financial impact of that order-size revision is too small to be of practical value. This insignificance of the financial impact can be best clarified through a numerical example.

4. A Numerical Example When $S_2 \equiv S_1$

Consider a one-item, one-customer situation with
D_1 = Annual Demand = 10,000 units/yr,
S_1 = Buyer's order processing cost = \$100/order,
P_1 = Price per unit paid by the buyer = \$10/unit,
H_1 = Buyer's inventory holding cost = 20% per year (or \$0.20 per dollar of inventory per year).

Then, assuming no discounts, the buyer's economic EOQ as given by Monahan's equation (2) is $Q_1^* = 1000$ units/order, with the annual cost of purchase, ordering and carrying being given by Monahan's equation (3B) as $TIC(Q_1^*) = \$102{,}000$/yr.

Let us further assume that the vendor's manufacturing frequency and average inventory are independent of the buyer's order frequency. Let S_2 and H_2 be numerically identical to S_1 and H_1 respectively, and let M_2, the vendor's gross profit as a percent of sales, =30% (i.e., \$0.3 per dollar of sales). In this case, by Monahan's equation (8) the vendor's initial profit is $YNP_2 = \$29{,}000$. The optimal K, given by his equation (11), is

[1] Lal and Staelin (1984) assume such an independence. Independence of production scheduling from customer order processing is a more realistic assumption since customer order timing may not be deterministic, and even if it was, the vendor must also consider production schedules of other items in his inventory while deciding on the production schedule of the item of concern. Furthermore, such an assumption may be the key to the generalization of Monahan's model to the n-customer situation.

[2] Lal and Staelin suggest that even if manufacturing frequency is independent of customer order frequency, the customer's order size affects the vendor's revenue stream. Their numerical example, however, suggests that the magnitude of change in the vendor's total cost due to such a change in revenue stream is rather small. Lal and Staelin's calculations assume that all customers in a homogeneous group increase their order size at the same time. In reality, if that happened, during the transitional period the vendor may have to increase his production substantially. Thus, the impact is not only on the revenue stream, but also on the manufacturing cost stream, the net effect of which is likely to be far smaller than that estimated by Lal and Staelin. The more likely situation is where a few customers change their order size at a time and the change in order size is accompanied by a change in the timing of that order, so that some customers may order larger quantity but at a time *later* than their normal ordering time. With such a phased-in change in customer ordering policies, the effect on a vendor's cash stream or other inventory carrying costs is likely to be particularly small. A phased-in change is more consistent with Lal and Staelin's own argument in their footnote 3 that buyers place their orders independent of one another.

$K^* = 1.414$. The breakeven price discount, given by his equation (6), is $d_K(BE) = \$0.0121$/unit, and the vendor's revised annual net profit, given by his equation, (9), is $YNP_2^1 = \$29{,}171.78$. The maximum discount the vendor could ever offer as given by Monahan's equation (6^1) is $d_K^1 = \$0.0292$.

In short, Monahan's model suggests that the vendor should offer a discount of between 1.21¢ to 2.92¢ per unit for an item that normally sells for $10 per unit provided the buyer orders 1414 units per order instead of his EOQ of 1,000 units. Notice that the maximum discount in this case is less than $\frac{3}{10}$th of 1% of the price of the item. It is unlikely that a buyer who spends a total of $102,000 per year on the purchase and procurement of this item will consider such a small discount seriously. Similarly, the net increase in the vendor's profit ($171.78), if the buyer does order 1414 units per order (for a price of $9.9879 instead of $10 per unit), may not even pay for the cost of communicating the price discount!

Thus, when S_2 and S_1 are approximately equal, the financial impact of Monahan's discount model is too small to be of practical significance. This is not surprising due to the insensitive nature of the EOQ model, and this conclusion is not subject to the specific numerical example presented. Wilson (1977) points out that "The U-shaped cost curve is quite flat near the optimal point. A deviation of 10% from EOQ will only mean a $\frac{1}{2}$% of difference in set up and storage costs—and these are usually only a minor portion of the total costs of labor and materials." Lal and Staelin's (1984) work also confirms that a quantity discount based purely on inventory cost considerations will be too small to be of practical significance. Unfortunately, they do not seem to be aware of this. Like Monahan, Lal and Staelin also focus on buyer's and vendor's inventory-related costs to arrive at a (slightly different) model of optimal quantity discounts.[3] In a real life application of their model, Lal and Staelin find that the vendor's order processing costs are comparable to (actually, slightly lower than) the buyer's order costs. The application of their optimal quantity discount model seems to result in annual savings of $45,963 for the seller. This looks sizable until one recognizes that this saving is for an item with annual sales of $63 million, and is based on discounts ranging between $0.0038 to $0.034 per unit (for various buyer groups), for an item that normally sells for $35.00 per unit.[4]

To repeat, we find that if S_2 and S_1 are of similar magnitude, a quantity discount model focusing exclusively on inventory related costs is of little practical significance.

5. A Numerical Example When S_2 Is Substantially Greater Than S_1

Now, let us consider a situation when S_2 is substantially greater than S_1—the situation that Monahan characterizes as the "maximum action" situation. Let D_1, S_1, P_1, H_1, H_2, and M_2 have the same values as in §4. However, let

S_2 = vendor's order processing and manufacturing setup costs
= $800 per order. Assume further that this cost consists of $100 for order-processing and $700 for manufacturing set-up, and

R_2 = vendor's annual production capacity for the item,
= 15,000 units/year. (Note that R_2 must be $\geq D_1$.)

[3] Lal and Staelin's model has several of its own theoretical shortcomings (in addition to the many it shares with Monahan's work). For example, (1) Lal and Staelin define their variable H_s as "Seller's cost of capital per year," instead of defining it as "Seller's cost of capital for *one unit of inventory* for one year," which is the way they use it. (2) They assume (rather unrealistically) that H_b, the buyer's cost of holding one unit of inventory, is independent of the price paid for that unit. For another example of the shortcomings of their model see footnote 2 above.

[4] Lal and Staelin do not explicitly report the numerical values of the discounts they propose. However, one can easily calculate these numerical values on the basis of their Table 2. For example, the annual savings to a buyer in group 2 because of "prior discounts" (i.e. discounts made available to group 1) are $6.37. Since a buyer in group 2 buys 1658 units per year, the discount to group 1 must be (6.37/1658) = $0.0038 per unit. Similar calculations indicate that the total discount for groups 1 to 6 is $0.034 per unit.

First, let us assume that the vendor's order-processing as well as manufacturing frequency is the same as the buyer's ordering frequency. Then, in the no discount situation, Monahan's equation (8) would estimate the vendor's profit to be \$22,000, whereas the correct profit as given by my equation (3) is \$21,533 (since the vendor incurs an inventory carrying charge of \$467). By his original formulae, Monahan would recommend a discount between $d_k(BE) = 13¢$ per unit to $d_K^1 = 53¢$ per unit for an order quantity of 3,000 units (i.e., $K = 3$). Under the 13¢ discount, Monahan would estimate the buyer's total costs to be unchanged and the vendor's profits to be \$26,000 (a \$4000 increase). The use of my profit formula (3) shows that with a 13¢ discount, the increase in the vendor's profit is only \$3,067 (since the vendor's inventory carrying charges go up from \$467 to \$1400). With the 53¢ discount, Monahan's computation indicates that the vendor's profit remains at \$22,000. By my formula (4) above, the vendor's profit declines from \$21,533 to \$20,600.

The optimal K recommended by my formula (5) is 2.48, with the discount range of 9¢ to 41¢ per unit.[5] With the 41¢ discount (for an order quantity of 2,480 units), the vendor's profit remains at \$21,533, whereas with the 9¢ discount, the vendor's profit is \$24,708 (i.e. an increase of \$3,175). Note that \$24,708 is the *upper limit* on the vendor's profit since the buyer would want a discount greater than his breakeven point of 9¢ per unit.

In what follows, we show that *the vendor's profit will be greater than this upper limit if, instead of considering a quantity discount, he simply considered a better production lot-size policy.*

6. A Vendor Using a More Rational Lot Size Policy

So far, we have used Monahan's assumption that the vendor's frequency of manufacturing set-ups is same as the buyer's order frequency. Now, let us suppose that the vendor uses a policy producing nQ_1 units every time he produces, where $n \geq 1$, and n is an integer. As shown in the Appendix,[6] the vendor's average inventory will be

$$(Q_1/2)((n-1)-(n-2)D_1/R_2). \tag{7}$$

In selecting the optimal n, the vendor would want to minimize SI_{n2}, the sum of his annual production set-up costs and his annual inventory costs, given by (8) below (since his sales revenues and order processing costs are unchanged):

$$SI_{n2} = (N_1S'_2/n) + (Q_1/2)((n-1)-(n-2)D_1/R_2)H_2(1-M_2)P_1, \tag{8}$$

where S'_2 is the production set-up cost for the vendor (\$700 in our numerical example). Clearly, SI_{n2} will be minimized when

$$-(N_1S'_2/n^2) + (Q_1/2)(1-D_1/R_2)H_2(1-M_2)P_1 = 0. \tag{9}$$

Given that $Q_1 = \sqrt{(2D_1S_1)/(H_1P_1)}$ and $N_1 = D_1/Q_1$ it can be shown that

$$n^* = \sqrt{(H_1/H_2)(S'_2/S_1)(1/(1-M_2))(R_2/(R_2-D_1))}. \tag{10}$$

Notice that if H_1 is of the same order as H_2, and S'_2 is substantially greater than S_1, n^* is likely to be greater than 2 since both $(1/(1-M_2))$ and $(R_2/(R_2-D_1))$ are by definition greater than one. n^* will be large particularly if R_2, the production capacity of the vendor, is close to D_1, the demand of the buyer. In general, we expect the vendor's

[5] Note that even when we grant all of Monahan's conditions, and S_2 is eight times S_1, the discount we are talking about is 1% to 4% of the price of the item. Thus, inventory related costs alone do not seem to explain the 10% to 20% quantity-discounts one often sees in the market. Perhaps, they may be explained on the basis of marketing considerations.

[6] The author wishes to thank an anonymous *Management Science* referee for suggesting the elegant proof in the Appendix.

capacity to be close to the buyer's annual demand, for if it is not, the vendor will save a substantial amount either by cutting back his capacity or by finding new uses for the same. As such, we expect $n^* \geq 2$.

Thus, in general, the rational strategy for the vendor is to deviate from the policy of producing to the buyer's order and to produce n (≥ 2) multiples of his orders.[7] Again, the financial implications of such a strategy may be best explained through our numerical example.

In our example, $n^* = 5.48$. Rounding it to the nearest integer,[8] we get $n^* = 5$. With this value of n^*, we obtain $SI_{n2} = \$2{,}800$. Given that the vendor must still incur order processing costs of $100 each for the ten orders of the buyer, the vendor's profit is now $YNP_2 = \$30{,}000 - 1{,}000 - 2{,}800 = \$26{,}200$ per year.

But observe that this profit is substantially greater than the upper limit of $24,708 we established for his profit in §5, when the vendor was pursuing only a quantity discount strategy. As such, it is clear that *an optimal production lot-size policy is superior to the policy of optimal price discounts when S_2 is substantially greater than S_1* (a condition in which Monahan expected "maximum action" using his model).

Of course, there is no reason why a vendor could not use both the optimal production lot-size and the optimal quantity discount strategy. But then, the discount must be based primarily on the vendor's order processing cost (not including his manufacturing set-up cost). As discussed in §4, however, in that case, the magnitude of quantity discount is so small, it has very little practical value.

7. Conclusion

Monahan's one-item, one-customer model has serious shortcomings both in its assumptions and in its practical usefulness. Monahan assumes that a vendor's manufacturing frequency is the same as a buyer's order frequency, and that the vendor's inventory carrying costs are unaffected by the buyer's order size. While each of these assumptions is questionable, certainly both cannot be valid at the same time. Monahan's model must either account for the effect of larger order sizes on the vendor's inventory carrying charges, or assume that the vendor's manufacturing frequency is independent of the buyer's ordering frequency. If the vendor's inventory carrying charges are to be accounted for, Monahan's model must be corrected as shown in §2. On the other hand, if the vendor's manufacturing frequency is different from the buyer's order frequency, it may be reasonable *not* to account for the vendor's inventory carrying charges. However, in that case, Monahan's variable S_2 must be redefined to include the vendor's order-processing cost only, but not his manufacturing set-up cost.

Furthermore, if the vendor's manufacturing set-up costs are high (a situation where Monahan thinks his model is most useful), it is not economically beneficial for the vendor to match his production frequency with the buyer's order frequency. The vendor would save more money by considering an optimal production frequency than by considering an optimal quantity discount policy. Thus, one of the assumptions implicit in Monahan's model is not only unrealistic but also irrational from a vendor's point of view.

[7] This result is not new. Szendrovits and Drezner (1980) concluded that in a multi-stage production system, when the constraint that batch sizes must be the same at each stage is relaxed, the "added flexibility naturally results in lower cost. Unless no batch shipment is warranted at any stage, the cost resulting from this model is invariably lower than the cost generated by a model requiring the same batch sizes at each stage."

[8] Even if we take $n = 6$, the financial implications are the same. Note also that if we had used the standard production lot size model, the vendor's optimal lot size would have been 5,477. Thus, the fact that the buyer's demand is discrete does not make a significant difference for the vendor's lot size determination. One could use the simple lot size formula and round it to the nearest integer.

Finally, a numerical example presented in this note questions the practical usefulness of Monahan's approach of focusing on inventory related costs alone, and ignoring any marketing considerations, in determining a vendor's optimal quantity discount strategy. Our numerical example suggests that once Monahan's model is corrected for its assumptions, we may be talking about discounts of the order of $\frac{3}{10}$ to 1% of the price of an item. Clearly, that is too small a discount to be of serious concern to either the buyer or the vendor. In fact, neither Monahan's model nor Lal and Staelin's (1984) model promises to explain commonly observed quantity discounts such as 10% of the unit price.

Appendix. Vendor's Average Inventory Using a Lot Size of nQ_1 Units

Suppose that the vendor uses a policy of producing nQ_1 units every time he produces. Let n be an integer greater than or equal to 1. Since we assume that the buyer's order frequency is deterministically known to the seller, we can assume that the vendor will schedule his production start-up at a time T_0, such that the first Q_1 units are produced exactly by the day (T_1) they are to be shipped; with the remaining $(n-1)Q_1$ units produced continuously thereafter over another $(n-1)Q_1/R_2$ year, until time T_e. In Figure 1, the production time T_0T_e equals nQ_1/R_2, and the boundary $T_0ZA_nB_n$ represents the cumulative production by the vendor over one cycle T_0T_0'. Note that the total cycle time is nQ_1/D_1 year. The height $T_eZ = nQ_1$.

Let $T_1, T_2, \ldots, T_n$ be the points in time when the buyer's first, second, $\ldots$, nth orders (of Q_1 units each) are shipped. Then the distance $T_0T_1 = Q_1/R_2$. Also, $T_1T_2 = T_2T_3 = \cdots = T_{n-1}T_n = Q_1/D_1$. The step-ladder $T_0T_1A_1B_1A_2 \cdots B_{n-1}A_nB_n$ represents the cumulative quantity shipped by the vendor during the cycle. As such, at any point, the vendor's inventory is the difference between the boundary $T_0ZA_nB_n$ and the step ladder $T_0T_1A_1B_1 \cdots B_{n-1}A_nB_n$.

Note that the area of the triangle T_0ZT_e is

$$(1/2)(nQ_1)(nQ_1/R_2). \tag{A1}$$

The area of the rectangle $T_eZA_nT_n$ is

$$nQ_1((n-1)((Q_1/D_1)-(Q_1/R_2))). \tag{A2}$$

The sum of the areas of the rectangles $T_1A_1Y_1T_n, B_1A_2Y_2Y_1, \ldots, B_{n-2}A_{n-1}B_{n-1}Y_{n-2}$ is

$$Q_1((n-1)Q_1/D_1 + (n-2)Q_1/D_1 + \cdots + Q_1/D_1). \tag{A3}$$

That is,

$$(Q_1^2/D_1)((1/2)n(n-1)). \tag{A4}$$

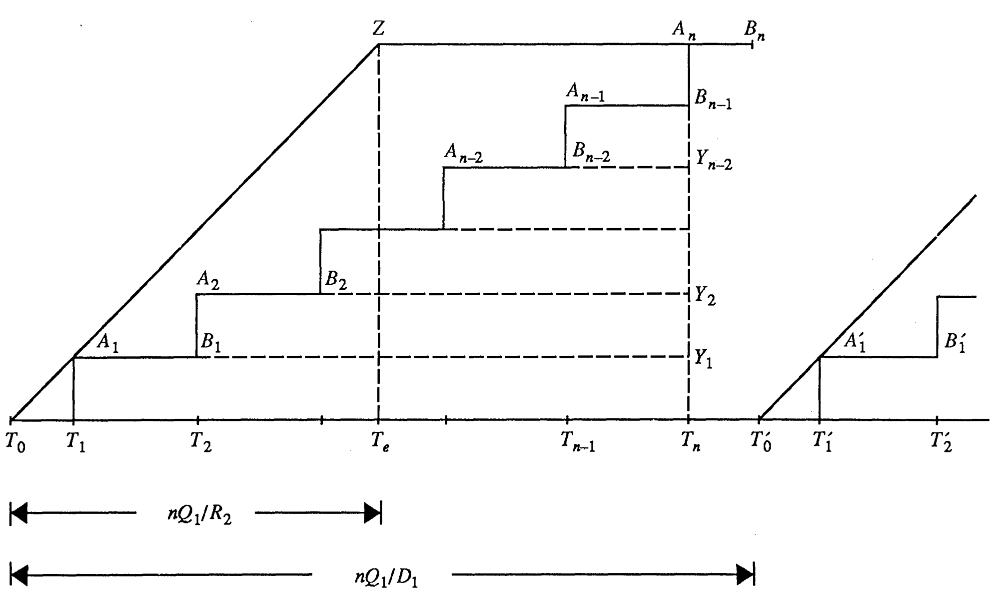

FIGURE 1. Cumulative Production and Shipments of a Vendor over a Cycle with Lot Size = nQ_1.

Adding (A1) and (A2) and subtracting (A4), the net area is

$$(nQ_1^2/2)((n-1)/D_1 - (n-2)/R_2). \quad \text{(A5)}$$

Dividing (A5) by the cycle time nQ_1/D_1, we have the vendor's average inventory as

$$(Q_1/2)((n-1) - (n-2)D_1/R_2) \quad \text{(A6)}$$

which is the same as formula (7) in the text.

References

LAL, R. AND R. STAELIN, "An Approach for Developing an Optimal Discount Pricing Policy," *Management Sci.*, (December 1984, p. 1524–1539.

MONAHAN, J. P., "A Quantity Discount Pricing Model to Increase Vendor Profits," *Management Sci.*, (June 1984), p. 720–726.

SZENDROVITS, A. Z. AND Z. DREZNER, "Optimizing Multi-Stage Production with Constant Lot Size and Varying Numbers of Batches," *Omega*, 8, 6 (1980), 623–629.

WILSON, G. T., "When Not To Use The Square Rule for EOQ," *Production and Inventory Management*, (Fourth Quarter 1977), 1–6.

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REPLY

ON "COMMENTS ON A QUANTITY DISCOUNT PRICING MODEL TO INCREASE VENDOR PROFITS"

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Professor Joglekar is to be thanked for his generally insightful Comments (1988) concerning my paper (Monahan 1984). He has in my view correctly assessed the limitations of two important production assumptions I made, assumptions which need to be rethought in any quantity-discount pricing models of the future. Yet, although I find myself in general agreement with Joglekar's comments, I wish to clarify some points concerning the model and its potential applicability.

I agree with Professor Joglekar that my model frequently will call for "small" quantity discounts from the supplier, and therefore does not adequately explain the many large discounts frequently observed in the world of business. But we should remember that the model was not built with this intention in mind. Quantity discounts are used here not to change the level of yearly demand, but merely to alter its current pattern. Larger objectives will doubtlessly call for larger price concessions. But we see here that even the more restrictive objective has potential economic value to the supplier and some, limited, negative financial implications for the buyer. "Small discounts" are this model's output, simply because this is what is called for to motivate the desired buying behavior. I don't believe we should expect otherwise.

Secondly, Professor Joglekar contends that an optimal production lot sizing strategy will outperform an optimal lot-for-lot/quantity discount strategy when $S_2 \gg S_1$. I believe that *for large enough* S_2/S_1 this is a provable mathematical result. If true, it tells us that *ultimately* we are better off abandoning this version of the quantity discount idea in favor of the more traditional lot sizing approach. Yet, Joglekar does not make clear at what

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