

AN EVALUATION OF FEDERAL POLICY INSTRUMENTS TO STIMULATE BASIC RESEARCH IN INDUSTRY*

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A model of the resource allocation behavior of a group of firms demonstrates that unaided industry allocation to basic (inappropriable) research is suboptimal. Furthermore, provision of seed money is generally counterproductive, while the provision of a matching subsidy is not very cost-efficient in increasing the allocation. Innovative policy alternatives are proposed. (RESEARCH AND DEVELOPMENT; GOVERNMENT; ENGINEERING; ECONOMICS)

1. Introduction

Economists have long contended that a free market system will fail to attain optimal allocation of resources to research and development (R&D). Arrow [1] has pointed out that a free enterprise system is likely to underinvest in R&D activities for the following two reasons:

First, since the purpose of any R&D activity is to generate new information, at the time the decision to invest in an R&D activity is made, the exact nature and value of the output are uncertain. That is, risk is inherent in R&D activities. Private firms often reject R&D activities that offer positive expected profit, because of the risk associated with these activities. On the other hand, society, as a whole, is likely to be relatively risk neutral. Hence, projects that are not acceptable to a private firm may be desirable from society's point of view.

Second, the benefits of the output of many R&D activities often cannot be fully captured by the firm or firms investing in those activities. Consider, for instance, the development and adoption of a pollution-free technology for steel processing in an underdeveloped economy where community awareness about industrial pollution is low and pertinent government regulation is nonexistent. (These were precisely the conditions that existed in the United States just a few decades ago.) Although such a development will help the steel producer improve his community relations and obtain some "goodwill", by and large the benefits of the improved process will accrue to the community at large, and the steel producer will not be able to obtain monetary payment for those benefits. This attribute of an R&D activity is called its "limited appropriability" or "inappropriability". Because of the limited appropriability of certain R&D activities, despite their social desirability, these activities may not be acceptable to a private firm.

Not all R&D activities possess the above two characteristics to the same degree. For example, basic research possesses a high degree of inappropriability in addition to some risk, whereas applied research and development activities are likely to be largely appropriable (under U.S. patent laws) but highly risky.¹ In any case, the existence of

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¹ The distinction between basic research (BR) and applied research and development (ARD) is not clear cut. Often activities that start out as pure scientific explorations turn out to have important (and patentable) applications. Similarly, activities aimed at specific applications often lead to important scientific discoveries that promise little opportunity for economic exploitation.

either one of the two characteristics is sufficient for the free market system to underinvest in an R & D activity.²

A literature review suggests that one way of correcting the underinvestment in either of the two types of R & D activities is to permit, if not promote, interfirm and/or industry-government cooperation. For example, Chamberlin [2], Hardin [5], Frohlich and Oppenheimer [3] have suggested that inappropriable goods may be provided adequately by a group of voluntarily cooperating firms. Samuelson [10] has shown that no matter how risky a project is, a risk averse but profit motivated firm would want some share of it, however small.

Unfortunately, research on the effectiveness and efficiency of specific federal policy instruments in stimulating different types of R & D activities is virtually nonexistent. In this paper, we provide an assessment of the effectiveness of some alternative policy instruments directed at stimulating cooperative basic research in industry. We restrict ourselves primarily to an evaluation of *fiscal instruments* for two important reasons:

(a) Other policy instruments (e.g., patent protection, legislated product or process standards, etc.) are likely to have a direct effect primarily on applied research and development and only an indirect, possibly negative, effect on basic research.³

(b) Fiscal instruments lend themselves to mathematical modeling with relative ease.

However, the modeling methodology described here can be used also to assess the effectiveness of certain nonfiscal instruments (e.g., government title policy in contract research, or the background patent release policy in cooperative research) that are

Even when an activity can be clearly identified as basic research, its benefits are not necessarily inappropriable. When the findings of a specific BR activity are kept completely secret, the potential benefits of that activity are not available to any one other than the investor. By the same token, not all ARD activities are appropriable. First, the results of ARD activities are not always patentable. Second, innovative imitators often succeed in circumventing patents. Third, the "pecuniary benefits" of an innovation (e.g., those that are available to a shrewd investor because of stock market fluctuations at the news of the innovation) are rarely recognized, much less realized, by the innovator himself. Finally, government regularity can curtail the appropriability of ARD activities under certain circumstances.

Consequently, we are not suggesting that BR activities are always inappropriable or that ARD activities are always appropriable. Yet, insofar as a firm engaging in a BR activity expects to share freely its findings with the scientific community at large, basic research is the closest commonly used term to refer to inappropriable R & D. Similarly, since a firm investing in an ARD activity does so with the hope of exploiting its results exclusively, ARD may be used to refer to appropriable R & D.

Strictly speaking, our models deal with R & D activities that are "characterized by inappropriability" versus those that are "characterized by appropriability". However, a constant repetition of such a reference to the type of R & D activity involved would be awkward. Instead, we refer to such activities by the commonly used term "basic research" and "applied research and development."

²Hirshleifer [6] has argued that because of the "pecuniary" benefits associated with new technical information, a free economy may *overinvest* in R & D. Pecuniary effects are defined as "the wealth shifts due to price reevaluations that take place upon release and/or utilization of information". These effects are "purely redistributive". That is, they may not add much social value, but for private firms they may be attractive insofar as they lead to the appreciation of assets or stocks of one firm at the expense of other firms. Hirshleifer argued that had Eli Whitney (inventor of the cotton gin) speculated upon the implications of his invention for the price of cotton and cotton bearing land, he could have realized benefits exceeding the value of the patent on the gin.

While Hirshleifer's argument sounds plausible, the fact is that Eli Whitney did not take advantage of the pecuniary benefits of his invention. More importantly, it seems doubtful that his investment of time and money in his invention was in anticipation of such pecuniary benefits.

We believe that R & D investment decisions are rarely made in anticipation of pecuniary benefits. In any event, the possibility of overinvestment in R & D activities does not negate the fact of underinvestment in many R & D activities.

³According to *Business Week*, July 3, 1978, a survey by the Industrial Research Institute indicates that because of regulations such as occupational safety and health, industrial research is becoming increasingly defensive and short run. The amount of basic research performed by industry had dropped from 38% of the national total in 1956 to 16% in 1976.

generally directed at the stimulation of applied research and development in industry [7].

In this paper we provide an outline of our methodology (§2), present one of our models (§3) and its findings pertinent to the effectiveness and efficiency of fiscal instruments aimed at stimulating basic research in industry (§4). The last section (§5) discusses our overall work (of which this paper is a small part) and gives several recommendations for broader government policies towards the stimulation of basic research in industry. In a second paper in this issue of *Management Science* (pp. 1016–1026), we discuss the stimulation of *applied* research.

2. Methodology

Fiscal instruments directed at the stimulation of basic research in industry can be categorized into two fundamental types:

(1) *The Provision of Matching Subsidies*: This category includes all government expenditures on basic research activities of an industry that are incurred in some proportion to the industry's own expenditures on these activities. An example of this type of expenditure is the special tax treatment offered to R&D expenditures. Alternatively, some federal grants for basic research may include a condition for the industry to match the grant money in some proportion.

(2) *Provision of Seed Money*: In this category we include all expenditures on the pertinent basic research activities that are incurred by the government without regard to whether the industry matches these expenditures in some proportion. Thus, this category includes basic research in government laboratories pertaining to the technology of an industry under consideration, or a research grant to a university or a private firm laboratory without a requirement for industry matching. Although such grants are often made in the hope that once some research findings are available, the industry will be sufficiently interested to carry research further (and are therefore called "seed money"), industry's contribution is not a precondition for government expenditure.

In evaluating the effectiveness and efficiency of these two types of fiscal instruments, we model the resource allocation behavior of a group of firms, each member of which stands to benefit from an investment in certain basic research activities. The characterization of basic research as inappropriable implies that a firm's benefits depend upon the total investment in the pertinent basic research activities from all sources (i.e., the firm itself, other firms in the industry, and the government) regardless of the identity of investors and the distribution of investments. Each firm is assumed to be rational, self-interested, risk averse and constrained by a maximum amount of investable resources at its disposal. It is assumed that funds that are not spent on pertinent basic research activities are expended on activities whose benefits are fully appropriable; that is, whose benefits are available to the investing firm and only to the investing firm. A firm's benefits from appropriable and inappropriable (i.e., basic research) activities are assumed to be independent of one another,⁴ but are each characterized by risk. That is, the benefits are assumed to be independent random variables with mean benefits from each type of activity being a function of only the investment in that type.

Each firm is assumed to seek a maximization of its expected utility from the combined benefits of its investment in appropriable activities and the total investment from all sources in inappropriable basic research. Thus, a firm's allocation of its own resources depends upon its perception of allocation by others. The firm may perceive

⁴Since there may be situations where the assumption of independence of benefits may be questionable, we did develop another model that assumed interdependence [7]. The results obtained were strikingly different but not contradictory to the results of the model presented in this paper.

the allocation of other firms to be totally independent of, or partly or fully dependent on, its own allocation. Similarly, government's allocation may involve the provision of seed money, a matching subsidy, or both.

Assuming that in the final equilibrium each firm's perception of others' behavior is actually borne out, we arrive at the industry's total allocation to the desired basic research activities under a variety of values for the two instruments of government intervention (including no intervention at all). This equilibrium allocation is seen as the actual allocation by the industry and is compared to the "socially optimum" allocation to basic research. The concept of Pareto optimality (which assumes that a resource reallocation that improves the utility of some firms without reducing the utility of any of the remaining firms is socially desirable, and that Pareto optimality is attained when no such reallocation is feasible) helps define the social optimum.

This framework allows us not only to confirm Arrow's notion that in the absence of government intervention an industry's allocation to basic research will be suboptimal, but also to assess the effectiveness and efficiency of alternative policy instruments. A policy instrument is said to be effective if it increases the industry's allocation to basic research without letting the total allocation surpass the socially optimum level. When two or more effective policy instruments are available (which may involve a variety of combinations of seed money and matching subsidy), they may be compared against one another on the basis of their relative efficiency. The efficiency of an instrument is defined as the ratio of the net increase in the industry's investment induced by the instrument to the government cost in implementing that instrument. Using these definitions of effectiveness and efficiency, we find that the provision of seed money is generally not an effective instrument of government intervention and that the provision of matching subsidies is not very cost-efficient. A theoretically ideal policy instrument is constructed and discussed.

3. A Model of Industry's Resource Allocation Behavior

3.1.1. *The Firm, the Industry Group and Investment Opportunities*

Consider the firm i belonging to a group $G \equiv \{1, 2, \dots, i, \dots, g\}$ of g firms. Let i have two investment opportunities X_i and Y . Assume that the benefits B_{X_i} of an investment in X_i accrue only to the firm i (i.e., X_i represents a purely appropriable investment opportunity for firm i). As such, only i is expected to invest an amount x_i in X_i . Let B_{X_i} be distributed exponentially⁵ as

$$f(B_{X_i}) = \lambda_{X_i} \exp(-\lambda_{X_i} B_{X_i}). \quad (1)$$

⁵The choice of the exponential distribution is rather arbitrary. It is selected primarily because of the algebraic convenience it offers. On the other hand, R&D managers would probably approve of the following properties of the exponential distribution in representing anticipated benefits of R&D investments: (a) the exponential distribution rules out the possibility of "negative benefits," (b) it assumes that the likelihood of a particular value of benefits decreases as the value increases, with the most likely value (i.e., the mode of the distribution) being zero. However, an exponential distribution also implies that the standard deviation of the distribution is the same as its mean. Most R&D managers may question that implication, often suggesting that the "technical risk" is far smaller. However, considering the commercial risk as well, it seems that benefits of a number of real life R&D investments may be reasonably well represented by an exponential distribution (particularly in the context of our model, since we have imposed only the most reasonable restrictions on the values of the parameters of the distribution).

In any case, we can report that we duplicated our analysis here assuming that benefits are distributed normally rather than exponentially [7]. The properties of a normal distribution differ substantially from those of an exponential distribution. A normal distribution does not rule out negative values, assumes that benefits are symmetrically distributed around the mean (and the mode) of the distribution, and allows for different values and for the mean and the standard deviation. Given these differences, we are greatly encouraged by the fact that most of our conclusions using a normal distribution are parallel, if not identical, to those using an exponential distribution.

The mean and standard deviation of this distribution are $1/\lambda_{X_i}$. We assume that the larger the investment firm i makes in X_i the larger are its mean benefits and that the relationship is linear.⁶ Therefore,

$$1/\lambda_{X_i} = \alpha_{X_i} + \beta_{X_i}x_i \tag{2}$$

where α_{X_i} and β_{X_i} are appropriate nonnegative constants. α_{X_i} represents the general competitive advantage of firm i . β_{X_i} is the increment in mean benefit per dollar increase in investment in X_i .

Let Y represent the basic (inappropriable) research activities whose benefits fall on the entire group G regardless of who invests in Y . Let y_i be firm i 's investment in Y . Let y_{G-i} be the investment in Y by all firms other than i . That is,

$$y_{G-i} = \sum_{j \neq i} y_j, \quad j \in G. \tag{3}$$

Hence, y_G the group's total investment in Y will be

$$y_G = y_i + y_{G-i}. \tag{4}$$

Further, let y_E be the investment in Y from sources external to the industry (e.g., government). Then the total investment y_T in Y from all sources will be

$$y_T = y_E + y_G = y_E + y_i + y_{G-i}. \tag{5}$$

Because of the inappropriability of Y , firm i 's benefits B_{Y_i} will depend upon this total investment y_T in Y . Let B_{Y_i} be exponentially distributed⁵ as

$$f(B_{Y_i}) = \lambda_{Y_i} \exp(-\lambda_{Y_i} B_{Y_i}) \tag{6}$$

the mean and standard deviation of which are $1/\lambda_{Y_i}$.

We further assume that basic research is a "lumpy" good. That is, in order for an investment in basic research to yield any meaningful results and consequent economic benefits, y_T must be at least equal to a minimum threshold level π . Let us, therefore, assume that

$$1/\lambda_{Y_i} = \begin{cases} \alpha_{Y_i} & \text{if } y_T < \pi, \\ \alpha_{Y_i} + \beta_{Y_i}y_T & \text{if } y_T \geq \pi, \end{cases} \tag{7}$$

where α_{Y_i} , β_{Y_i} and π are appropriate nonnegative constants. α_{Y_i} represents the a priori advantage of firm i because of past basic research in the industry. β_{Y_i} is the increase in mean benefits per dollar of investment in Y beyond the threshold level π .

3.1.2. The Utility Function and Risk Adjusted Benefits

Let firm i 's utility function be a linear transform of the exponential function

$$U_i(w_i + B_i) = (1/r_i)(1 - \exp(-r_i(w_i + B_i))) \tag{8}$$

⁶In a recent survey of literature, Kamien and Schwartz [8] report that "Schmookler found patents and R&D expenditures to be closely related . . . with 85 percent of the interindustry variation in patents pending explained by the variation in R&D expenditures . . . Scherer's regression analysis indicated a very nearly linear relationship between the number of R&D personnel in 1955 and the number of patents issued a firm in 1959 . . ."

Kamien and Schwartz go on to conclude that ". . . on average, a direct relation between innovational effort and innovational output exists. However, it is likewise true that the transformation may depend on factors other than effort, and it may not be linear."

Thus, Kamien and Schwartz provide some support for our assumption of linearly related costs and benefits and also raise some doubts about it. Given that available literature does not provide an alternative relationship between research inputs and outputs, we have assumed a linear relationship because of its mathematical simplicity.

where w_i is the initial wealth of firm i and r_i is the risk aversion constant. We assume $r_i > 0$. As Gupta and Cozzolino [4, pp. 249–252] point out, the exponential utility function offers several advantages. First, since we have assumed $r_i > 0$, the function U_i is strictly concave, and represents aversion towards risk. Second, the constancy of r_i in the exponential utility function implies that the premium a firm is prepared to pay in order to avoid the risk associated with the probable benefits is dependent only on the variance of the distribution and not on the expected total wealth of the firm either before or after a particular decision.⁷ We believe that these properties of the exponential utility function are quite realistic in a wide variety of situations. Using Gupta and Cozzolino's [4] formula for the certainty equivalent or risk adjusted value (RAV), we have

$$\text{RAV}(B_{X_i}) = (1/r_i)\ln[1 + r_i(\alpha_{X_i} + \beta_{X_i}x_i)] \quad \text{and} \tag{9}$$

$$\text{RAV}(B_{Y_i}) = \begin{cases} (1/r_i)\ln(1 + r_i\alpha_{Y_i}) & \text{if } y_T < \pi, \\ (1/r_i)\ln[1 + r_i(\alpha_{Y_i} + \beta_{Y_i}y_T)] & \text{if } y_T \geq \pi. \end{cases} \tag{10}$$

In most real life situations, we expect risk adjusted values (RAV) to be monotonically increasing functions of the investment in the pertinent opportunity. At the same time, we expect the law of diminishing marginal returns to hold. The reader can easily verify that the forms of benefit distributions and utility functions we have assumed do possess these expected characteristics.

3.1.3. The Rational Firm's Behavior

Now, if we assume that a firm's benefits from investments in X_i and Y are independent of one another,¹ a rational firm would want to maximize

$$F_i = \text{RAV}(B_{X_i}) + \text{RAV}(B_{Y_i}) \tag{11}$$

$$\text{subject to } x_i + y_i = R_i \tag{12}$$

where R_i represents the total investable resources at the disposal of firm i .

Note that if $y_T < \pi$

$$F_i = (1/r_i)[\ln(1 + r_i\alpha_{X_i} + r_i\beta_{X_i}x_i) + \ln(1 + r_i\alpha_{Y_i})] \tag{13}$$

and if $y_T \geq \pi$

$$F_i = (1/r_i)[\ln(1 + r_i\alpha_{X_i} + r_i\beta_{X_i}x_i) + \ln(1 + r_i\alpha_{Y_i} + r_i\beta_{Y_i}y_T)]. \tag{14}$$

Since F_i depends on the total investment y_T in Y rather than the investment y_i by firm i alone, it is imperative that a rational firm make certain assumptions about the resource allocation behavior of other firms and that of the government. Following Cournot's work on oligopolies, economists often assume that firm i will believe that other firms' behavior is not influenced by its own. That is, y_{G-i} is independent of y_i . There is, however, another school of thought that assumes that each firm believes that other firms will match, positively or negatively, its own investment in Y . That is, they assume that y_{G-i} is an increasing (if the matching is positive) or a decreasing (if the matching is negative) function of y_i . In order to build a general model that can account for either of these two assumptions, we shall say that

$$y_{G-i} = k_{G-i} + m_{G-i}y_i \tag{15}$$

⁷Note that we have not assumed that all firms are equally risk averse. Different firms will have different values for r_i .

where k_{G-i} is a nonnegative constant representing the minimal investment in Y by all firms other than i , no matter what i does ($k_{G-i} \geq 0$), and m_{G-i} is a constant representing their matching rate per dollar of i 's investment in Y . The constant m_{G-i} can take on a positive, zero or negative value depending on the size, structure and cooperative (or exploitative) climate in an industry. But m_{G-i} is always greater than -1 .

Furthermore, if the government believes that in the absence of its encouragement, industry's investment in the basic research will be suboptimal, government may provide either seed money or matching subsidy, or both. Government's investment y_E in Y can be represented as

$$y_E = k_E + m_E y_G \tag{16}$$

where k_E is the government seed money regardless of industry's investment in basic research, and m_E is the matching government subsidy per dollar of industry investment. We expect $k_E \geq 0$ and $m_E \geq 0$.

Given the foregoing assumptions, the total investment y_T in Y can be written as

$$y_T = k_E + (1 + m_E)k_{G-i} + (1 + m_E)(1 + m_{G-i})y_i. \tag{17}$$

With these parameters defined, we can now turn to understanding the behavior of a rational member of our group. As pointed out earlier, firm i 's problem is to maximize F_i subject to its resource constraint. If firm i expects that y_T will be less than π in the final equilibrium, then its F_i is defined by (13). Since (13) is independent of y_i but is an increasing function of x_i it will be maximized at

$$x_i^* = R_i, \quad y_i^* = 0, \tag{18}$$

where x_i^* and y_i^* are the optimal values of x_i and y_i .

We shall, for the moment, ignore this corner point solution and assume that for every firm in our group, there is a noncorner optimal solution. The noncorner solution for firm i will be obtained by maximizing (14) subject to (12). Substituting (17) in (14) we can rewrite the objective function as

$$F_i = (1/r_i)\ln(1 + r_i\alpha_{X_i} + r_i\beta_{X_i}x_i) + (1/r_i)\ln[1 + r_i\alpha_{Y_i} + r_i\beta_{Y_i}(k_E + (1 + m_E)k_{G-i} + (1 + m_E)(1 + m_{G-i})y_i)]. \tag{19}$$

It is well known that in such a constrained maximization problem, if there is a noncorner solution, it satisfies the condition

$$\partial F_i / \partial x_i = \partial F_i / \partial y_i \tag{20}$$

and the resource constraint (12).

Thus, after simplification, we see that the optimal allocations x_i^* and y_i^* are as below:

$$\left. \begin{aligned} x_i^* &= \frac{1}{2}(R_i + V_{Y_i} - V_{X_i}), \\ y_i^* &= \frac{1}{2}(R_i + V_{X_i} - V_{Y_i}), \end{aligned} \right\} \tag{21}$$

where V_{Xi} and V_{Yi} are abbreviations for the following expressions

$$V_{Xi} = (1 + r_i \alpha_{Xi}) / (r_i \beta_{Xi}), \tag{22}$$

$$V_{Yi} = \frac{1 + r_i \alpha_{Yi} + r_i \beta_{Yi} (k_E + k_{G-i} (1 + m_E))}{r_i \beta_{Yi} (1 + m_E) (1 + m_{G-i})}. \tag{23}$$

Note that, under our assumptions,

$$V_{Xi} \geq 0 \quad \text{and} \quad V_{Yi} \geq 0. \tag{24}$$

It follows that in order for x_i^* and y_i^* to be both positive

$$R_i > |V_{Xi} - V_{Yi}| \quad \text{and} \tag{25}$$

$$y_T \geq \pi. \tag{26}$$

3.1.4. The Aggregate Allocation to Y in Industry Equilibrium

Although all resource allocation decisions of every firm are not optimal, in studying aggregate industry behavior it seems reasonable to assume that firms are individually rational and that, by and large, they do conform to choices dictated by individual firms' optimal conditions. Thus, we assume that the foregoing conditions (11) to (26) are descriptive of the actual behavior of firm i . We use this generalized solution to determine the total industry's allocation of resources to the joint investment opportunity Y . We make one further assumption, namely that each firm's perception of other firms' strategy is borne out by the equilibrium solution for the total industry, i.e., a point from which no individual has any incentive to depart. In other words, we assume that firm i would adjust its perceptions (values of k_{G-i} , m_{G-i}) until they explain the actual investment, y_{G-i} , in Y by all firms other than i . That is,

$$k_{G-i} + m_{G-i} y_i^* = y_{G-i}^*. \tag{27}$$

Hence,

$$k_{G-i} = y_G^* - (1 + m_{G-i}) y_i^* \tag{28}$$

where y_i^* and y_G^* are optimal investments of firm i and the group G respectively, given by conditions (11) to (26) as appropriate. In the equilibrium we can rewrite V_{Yi} in (23) as

$$\begin{aligned} V_{Yi} &= \frac{1 + r_i \alpha_{Yi} + r_i \beta_{Yi} (k_E + [y_G^* - (1 + m_{G-i}) y_i^*] (1 + m_E))}{r_i \beta_{Yi} (1 + m_E) (1 + m_{G-i})} \\ &= \frac{W_{Yi}}{(1 + m_E) (1 + m_{G-i})} + \frac{k_E}{(1 + m_E) (1 + m_{G-i})} + \frac{y_G^*}{(1 + m_{G-i})} - y_i^* \end{aligned} \tag{29}$$

where

$$W_{Yi} = (1 + r_i \alpha_{Yi}) / (r_i \beta_{Yi}). \tag{30}$$

Consequently, we can rewrite y_i^* in (21) as

$$y_i^* = R_i + V_{Xi} - \frac{W_{Yi}}{(1 + m_E) (1 + m_{G-i})} - \frac{k_E}{(1 + m_E) (1 + m_{G-i})} - \frac{y_G^*}{(1 + m_{G-i})}. \tag{31}$$

Assuming that all firms have noncorner solutions, adding their y_i^* , and simplifying, we

get

$$y_G^* = \frac{1}{1 + \mu} \left(R_G + V' - \frac{W'}{1 + m_E} - \frac{\mu k_E}{1 + m_E} \right) \quad \text{where} \quad (32)$$

$$\left. \begin{aligned} \mu &= \sum_i 1/(1 + m_{G-i}), \\ V' &= \sum_i V_{Xi} \quad \text{and} \\ W' &= \sum_i W_{Yi}/(1 + m_{G-i}). \end{aligned} \right\} \quad (33)$$

Note that under our assumptions, μ , V' , and W' will each have nonnegative values. y_T^* , the total equilibrium investment in Y (including governments' investment), will be given by $y_T^* = k_E + (1 + m_E)y_G^*$ i.e.,

$$y_T^* = [1/(1 + \mu)] [k_E + (1 + m_E)(R_G + V') - W']. \quad (34)$$

Therefore, for condition (26) to hold, the right-hand side of (34) should exceed π . Similarly, the equilibrium investment of every firm can be obtained by substituting the value of y_G^* as obtained from (32) into (31).

It is clear that given the values of the various parameters in (31) to (34) there is one and only one equilibrium solution for each firm.

Note that one of the original parameters, k_{G-i} , is eliminated in the final equilibrium. The reason is that by condition (28) a given value of m_{G-i} uniquely determines the equilibrium k_{G-i} (and vice versa). The important thing to note is that the condition $k_{G-i} \geq 0$ must still be valid. By (28) that condition may now be written as

$$(1 + m_{G-i}) \leq y_G^*/y_i^*. \quad (35)$$

The importance of this condition is evident in the following two sections.

3.2.1. The Group Rational Investment in Y

So far, we have discussed the individually rational equilibrium solution to investment in Y . That is, we assumed that each firm decided its allocation of resources purely on its own. We assumed that even if there was a coordinating mechanism, the ultimate choice was still left to the individual firm. Instead, let us now assume that the group has a "master planner" (a benevolent dictator) who takes the "Pareto view" that every firm should be as well off as it can be, without jeopardizing the benefits of other firms in the group. Such a master planner would require [11, pp. 543-544] that members of the group share every marginal dollar of investment in Y in proportion to the marginal increases in their respective RAVs. That is,

$$\partial F_i/\partial y_i = \partial F_j/\partial y_j = \partial F_G/\partial y_G = \partial (\sum F_j)/\partial y_G \quad \text{for all } i, j. \quad (36)$$

We can rewrite this condition as

$$\frac{\beta_{Yi}(1 + m_E)(1 + m_{G-i})}{1 + r_i\alpha_{Yi} + r_i\beta_{Yi}y_T} = \sum_{j \in G} \frac{\beta_{Yj}(1 + m_E)}{1 + r_j\alpha_{Yj} + r_j\beta_{Yj}y_T}. \quad (37)$$

That is,

$$\frac{\beta_{Yi}(1 + m_E)}{1 + r_i\alpha_{Yi} + r_i\beta_{Yi}y_T} = \left(\frac{1}{1 + m_{G-i}} \right) \sum_{j \in G} \frac{\beta_{Yj}(1 + m_E)}{1 + r_j\alpha_{Yj} + r_j\beta_{Yj}y_T}. \quad (38)$$

Adding the two sides of (38) for all $i \in G$, and eliminating the common factor, we get

$$\sum_{i \in G} [1/(1 + m_{G-i})] = 1. \tag{39}$$

It can be shown that if each firm adheres to the prescribed sharing rule (hence, if the above condition is satisfied) and if each firm maximizes its RAV, then at the industry equilibrium the perceived k_{G-i} for each i will be zero.

To verify this, note that by (28)

$$y_G^*/(1 + m_{G-i}) - y_i^* = k_{G-i}/(1 + m_{G-i}). \tag{40}$$

Adding for all i , we have

$$y_G^* \sum_i [1/(1 + m_{G-i})] - y_G^* = \sum [k_{G-i}/(1 + m_{G-i})]. \tag{41}$$

Using (39) we can see that the left-hand side of (41) is zero. Since $m_{G-i} > -1$ for all i and since we expect a finite matching rate, it follows that every k_{G-i} must be zero. *In other words, Pareto optimality will result when each member perceives a pure matching behavior on the part of other members.*

From (32), (33) and (39) it can be seen that if there is no government intervention (i.e., if $k_E = m_E = 0$), the group rational investment y_G^{**} for the industry would be

$$y_G^{**} = \frac{1}{2} \left[R_G + V' - \sum_{i \in G} \{ W_{Yi}/(1 + m_{G-i}^{**}) \} \right] \tag{42}$$

where m_{G-i}^{**} is the *Pareto optimal matching rate* as enforced by the master planner. V' was defined in (33) and W_{Yi} in (30). We have established in (40) and (41) that this matching rate is such that it reflects pure matching behavior on the part of all the members. That is

$$k_{G-i}^{**} = 0 \quad \text{for all } i. \tag{43}$$

Consequently,

$$1 + m_{G-i}^{**} = y_G^{**}/y_i^{**} \quad \text{for all } i. \tag{44}$$

Note then, from condition (39), that under Pareto optimality the perceived matching rate attains its maximum value for each firm. That is,

$$m_{G-i}^{**} \geq m_{G-i} \quad \text{for all } i, \tag{45}$$

where m_{G-i} represents any value for others' matching rate that i can feasibly perceive. Condition (45) leads us to an important conclusion in the following section.

3.2.2. *The Existence of Suboptimality in the Absence of Government Intervention*

When there is no government intervention (i.e., $k_E = m_E = 0$), the individually rational group investment y_G^* is given by the modified form of (32),

$$y_G^* = [1/(1 + \mu)](R_G + V' - W'). \tag{46}$$

Now, because of conditions (42) and (45), it is obvious that

$$y_G^* \leq y_G^{**}. \tag{47}$$

In other words, the *individually rational* group investment in Y can never exceed the *group rational* group investment in Y .

It might seem at first glance that if some cost sharing rules other than the one prescribed by the master planner lead to suboptimal group investment (compared to the Pareto level of investment), some other cost sharing rules would lead to supraop-

timal investment in Y . The result in (47) negates such a proposition. If there is to be an industry equilibrium, there cannot be supraoptimal investment in Y under any feasible cost sharing rule for our model.

We contend further that, in general, individually rational group investment will be substantially less than the Pareto optimal group investment in basic research for several reasons including:

(a) Most industry groups (including those that are quite organized) are primarily voluntary, and do not possess a “benevolent master planner” capable of enforcing his dictates. In a voluntary group, Cournot’s assumption is likely to hold, particularly if the group is large. That is, pure matching behavior is unlikely.

(b) Even in organized groups, because of the “free-rider problem”⁸ associated with an inappropriable activity such as basic research, it is in the interest of a firm to declare its marginal benefits from Y to be smaller than what they are, hoping that others would be more truthful and that their investment in Y will be proportionately greater. It can also be shown with the use of the objective function (14) and the cost constraint (12) of this model that such a strategy leads to a higher value of F_i for i provided others are truthful (we do not show this here because the algebra is rather cumbersome). When every firm adopts such a free-rider strategy, however, the investment in Y , if any, is highly suboptimal.

Thus, we have established that in voluntary industry groups the individually rational group investment in basic research will be suboptimal in the absence of government intervention. Our analysis so far contradicts the suggestions of Chamberlin [2], Hardin [5] and Frohlich and Oppenheimer [3] that voluntarily cooperating industry groups may invest socially optimum amounts in inappropriable activities such as basic research. Instead, it supports Olson’s [9] position that such voluntary groups will fail to allocate adequate resources to inappropriable activities. It strengthens Arrow’s [1] conclusion that a free enterprise system will invest suboptimally in basic research, by indicating the truth of that proposition even when firms in an industry are allowed to cooperate with one another on a voluntary basis. Clearly, there is a legitimate need for government intervention to promote industry cooperation for investment in basic research.

4. Analysis and Findings

So far, we have built a general model to describe the behavior of an industry group as it allocates its resources between basic research and other investment opportunities. We have allowed for complete heterogeneity among firms, for a variety of perceptions of other firms’ behavior, and for the possibility of the need for a threshold investment in basic research.

This general model can be manipulated to analyze the effects of several relevant factors upon an industry’s allocation of resources to basic research, particularly in relation to the Pareto optimal level of allocation. Elsewhere [7], we have carried out an assessment of the effects of the following factors: (i) the relative benefits associated with the two investment opportunities, (ii) the investable resources of each firm, (iii) the cooperative or exploitative attitudes of the industry (i.e., the perceived matching rate), (iv) the degree of risk aversion on the part of each firm, and (v) the structure of the industry concerned (both the number of firms involved and their relative sizes).

Here we present only our assessment of effectiveness and efficiency of the following

⁸The “free rider problem” is an economist’s term to describe this situation. For an extensive discussion of this problem, see Olson [9].

two government policy instruments:

- (a) the provision of a matching subsidy, m_E , and
- (b) the provision of “seed money”, k_E .

We assume that government’s concern is to bridge the gap between industry’s actual investment in basic research [y_G^* as given by (46)] and its group rational investment [y_G^{**} as given by (42)] in the absence of any government intervention.

With government intervention, the total investment in Y is given by (34). Let us designate that by $y_{T|E}^*$ to emphasize the presence of government intervention. Thus, government wants $y_{T|E}^*$ to be close to y_G^{**} of (42). One way of accomplishing that is by having government spend the desired amount all by itself. However, government’s real objective is to increase the industry’s investment as much as possible, without creating an unwarranted burden in terms of tax dollars. Equation (32) gives the industry’s investment in Y , under government intervention, which we represent by $y_{G|E}^*$ to emphasize the presence of intervention. $y_{G|E}^* - y_G^* = N_{G|E}$ gives the net effect of government intervention on the industry’s investment. We shall say that an instrument is *effective* only if $N_{G|E}$ is positive, and the corresponding $y_{T|E}^*$ does not exceed y_G^{**} .

When we have two or more effective instruments, we shall be further concerned with the *efficiency* (e) of the tax dollar under each. The cost of a measure to the government is $C_E = y_{T|E}^* - y_G^*$. The ratio $e = N_{G|E}/C_E$ will measure the efficiency of an instrument.

4.1. The Effectiveness of Seed Money

If we assume that the benefit function, risk aversion and perceived cooperation in an industry are not affected by the presence of government intervention, differentiating $y_{G|E}^*$ w.r.t. k_E we get

$$\partial y_{G|E}^* / \partial k_E = -\mu / [(1 + m_E)(1 + \mu)]. \tag{48}$$

Under our assumptions, this is negative. Thus, when all firms have a noncorner solution, industry’s investment in Y will be a decreasing function of the seed money provided by the government. Since $N_{G|E}$ in this case will be negative, this implies that provision of seed money is not an effective measure of government intervention. In fact, we must say that *the provision of seed money is counterproductive* since it reduces the industry’s investment. Intuitively, this occurs because as government increases its investment in basic research, firms feel freer to allocate their own funds primarily to activities whose benefits are more appropriable.

It can be shown that what is true about k_E ’s effectiveness in the all-noncorner-solution case is also true when some firms have a corner solution while others have noncorner solutions.⁹

There is one case, however, where k_E is *effective*. This is the situation when Y is lumpy, and in the absence of government intervention, condition (26) is not satisfied, although (25) is. In this case,

$$y_G^* = 0. \tag{49}$$

However, because

$$\partial y_{T|E}^* / \partial k_E = 1 / (1 + \mu) \tag{50}$$

is positive, with positive value for k_E , government could see to it that

$$y_{T|E}^* \geq \pi \quad \text{and} \tag{51}$$

$$y_{G|E}^* > 0. \tag{52}$$

⁹For a discussion of this case, see [7].

In fact, in this case, if government wants to use the instrument of seed money alone, ($k_E > 0, m_E = 0$) its optimal choice for the value of k_E would be such that

$$y_{T|E}^* = [1/(1 + \mu)][k_E + R_G + V' - W'] = \pi. \tag{53}$$

Beyond this point, a marginal increase in k_E would reduce the industry's investment in Y . Under this optimal choice, the cost to the government would be

$$C_E = k_E = (1 + \mu)\pi - R_G - V' + W', \tag{54}$$

and the net increase in the industry's investment would be

$$\begin{aligned} N_{G|E} &= y_{G|E}^* - y_G^* \\ &= y'_{G|E} \quad \text{since } y_G^* = 0. \end{aligned} \tag{55}$$

Using (32) and remembering $m_E = 0$, we have

$$N_{G|E} = [1/(1 + \mu)][R_G + V' - W' - \mu k_E]. \tag{56}$$

Using the value of k_E in (54), therefore,

$$N_{G|E} = R_G + V' - W' - \mu\pi. \tag{57}$$

Therefore, the efficiency of the proposed seed money is

$$e = \frac{N_{G|E}}{C_E} = \frac{R_G + V' - W' - \mu\pi}{(1 + \mu)\pi - R_G - V' + W'}. \tag{58}$$

This efficiency can be compared with the efficiency of other effective instruments for the lumpy case.

Thus, except when Y is lumpy (i.e., $\pi > 0$) provision of seed money is not an effective measure of government intervention if we assume that such a provision has no effect on the benefit functions, risk aversion and perceived cooperation in an industry.

We must admit the possibility that with the seed money provided by the government, an industry may set up a consortium to promote an atmosphere of cooperation and a mechanism for collecting and investing the allocations of member firms to the joint investment opportunity Y . If this means increased values of m_{G-i} for individual members, then the use of seed money may turn out to be effective even in cases other than the lumpy case. However, we contend that as long as the consortium maintains a voluntary posture (or does not assume a tax authority) because of the free rider problem, in the equilibrium, the matching rate will tend to be much smaller than the Pareto optimal matching rate. Seed money will have little, if any, effect on m_{G-i} , and consequently on the suboptimality of industry investment in Y .

4.2. The Effectiveness of Matching Subsidies

Assuming that the benefit functions, risk aversion and perceived cooperation in an industry are not affected by the existence of government intervention in the all-non-corner-solution case, the partial derivative of $y_{G|E}^*$ w.r.t. m_E is

$$\frac{\partial y_{G|E}^*}{\partial m_E} = \frac{1}{(1 + m_E)^2} \cdot \frac{1}{(1 + \mu)} (W' + \mu k_E) \tag{59}$$

which, under our assumptions, is positive.

That is, m_E is effective in inducing additional investment from the industry. It can be verified easily that the second derivative $\partial^2 y_{G|E}^* / \partial m_E^2$ is negative. This means the marginal investment is a decreasing function of the marginal increase in m_E .

Assuming government is considering the use of only the matching subsidy (i.e., $k_E = 0$, $m_E > 0$), the net increase in industry's investment with a given m_E will be

$$\begin{aligned} N_{G|E} &= y_{G|E}^* - y_G^* \\ &= \frac{m_E}{(1 + \mu)(1 + m_E)} W' \end{aligned} \quad (60)$$

while the cost of the instrument to the government will be

$$\begin{aligned} C_E &= m_E y_{G|E}^* \\ &= \frac{m_E}{(1 + \mu)} \left[R_G + V' - \frac{1}{(1 + m_E)} W' \right]. \end{aligned} \quad (61)$$

Hence, the efficiency of a matching subsidy, in the all-noncorner-solution case will be

$$e = \frac{N_{G|E}}{C_E} = \frac{W'}{(1 + m_E)(R_G + V') - W'}. \quad (62)$$

Numerical calculations (see Appendix), with reasonable values for the parameters involved, indicate that *the cost efficiency of a matching subsidy is often quite low*. Particularly when government chooses a matching rate that would completely eliminate suboptimality of investment (i.e., such that $y_{T|E}^* = y_G^*$), cost efficiency is a small fraction. In other words, at that level, government would spend many times the amount of increase it would induce in the industry's investment in Y . The most important reason underlying this low efficiency value is that our model assumes a matching subsidy for the *entire investment* of the industry, including y_G^* , the investment industry would have made even without the subsidy. This is how a matching subsidy is currently employed in practice. The analysis indicates, however, that government should propose a *marginal matching* subsidy that would be available only *beyond a minimal level* (y_G^*) of an investment on the part of the industry.¹⁰

In short, *a matching subsidy, as it is currently provided, is an effective but not very efficient measure of government intervention* for the all-noncorner-solution case. This is also true as long as some firms are operating at noncorner points in the absence of government intervention.

When the investment opportunity is lumpy, and condition (26) is not satisfied without government intervention, both the provision of seed money and the provision of a matching subsidy are effective instruments of intervention. Let us now compare the efficiencies of these two measures in the lumpy case. We have already defined the optimal k_E and its efficiency in 4.1. For a comparison, we now consider that, with $k_E = 0$, government chooses a value of m_E such that

$$y_{T|E}^* = \frac{(1 + m_E)(R_G + V') - W'}{(1 + \mu)} = \pi. \quad (63)$$

¹⁰The 1981 Reagan tax plan proposed a special tax write-off for R&D investment of a firm that exceeds its average R&D investment during the prior three years. Our analysis supports this type of "matching subsidy" beyond a minimal level.

That is,

$$m_E = \frac{(1 + \mu)\pi + W' - R_G - V'}{R_G + V'}. \quad (64)$$

In this case,

$$\begin{aligned} N_{G|E} &= y_{G|E}^* - y_G^* \\ &= y_{G|E}^* \quad \text{since } y_G^* = 0, \end{aligned} \quad (65)$$

while the cost to the government is

$$C_E = m_E y_{G|E}. \quad (66)$$

Consequently, the efficiency of m_E is given by

$$e = \frac{N_{G|E}}{C_E} = \frac{1}{m_E} = \frac{R_G + V'}{(1 + \mu)\pi + W' - R_G - V'}. \quad (67)$$

From (58) and (67) it is easy to see that in the lumpy case *provision of a matching subsidy is a more efficient instrument than the provision of seed money.*

Of course, it should be reiterated that, in all of this analysis, we have assumed that government intervention does not affect the perceived intra-industry matching rate (m_{G-i}). In some cases, seed money may effect m_{G-i} . However, it is unlikely that the availability of government matching subsidies would alter intra-industry matching behavior. With that caveat in mind, we conclude that, in general, provision of a matching subsidy is a more effective and efficient instrument of government intervention than provision of seed money for reducing suboptimality of investment in basic research.

4.3. Construction of an Optimal Instrument of Government Intervention

In the foregoing analysis, we assumed that government used only *nonnegative inducements* ($k_E \geq 0, m_E \geq 0$) to encourage the industry to increase its investment in Y . The fact that provision of seed money is often counter-productive (i.e., $\partial y_{G|E}^* / \partial k_E$ is negative), however, suggests that if government were to choose “negative seed money” (e.g., a flat tax), then industry’s investment in Y may increase. On the other hand, such a tax would reduce the total investment (after subtracting the tax from the industry’s allocation to Y) since $\partial y_{T|E}^* / \partial k_E$ is positive. Thus, a flat tax alone will not solve the problem of suboptimality of investment in basic research.

But we can construct an alternative involving a flat tax and a matching subsidy such that both the industry’s allocation to Y and the total investment in Y would be positively affected. In fact, assuming that a government is not averse to a benevolent use of its taxing authority, we can construct an instrument (with $k_E < 0, m_E > 0$), such that the “net tax” to the industry (after the matching) is zero. The condition for this is

$$k_E + m_E y_{G|E}^* = 0. \quad (68)$$

In this case,

$$y_{T|E}^* = k_E + (1 + m_E) y_{G|E}^* = y_{G|E}^* \quad \text{and} \quad (69)$$

$$C_E = 0. \quad (70)$$

$$\text{Since } \partial y_{G|E}^* / \partial k_E < 0 \text{ and } \partial y_{G|E}^* / \partial m_E > 0, \quad (71)$$

$$\text{when } k_E < 0 \text{ and } m_E > 0, N_{G|E} \text{ will be } > 0. \quad (72)$$

Thus, the theoretical efficiency ($e = N_{G|E}/C_E$) of the measure will be infinite. Of course, even a benevolent or “paternalistic” use of government’s tax authority may be seen as objectionable by some. We have not accounted for the intangible costs that a democratic system may associate with the very use of government’s coercive power.

With proper choice of the values of k_E and m_E , government can raise industry’s allocation to Y as high as it wants. Let us assume that government chooses to take it to the Pareto optimal level for the industry, i.e.,

$$y_{G|E}^* = y_G^{**}. \quad (73)$$

Now, substituting (68) into (32) and simplifying, we have

$$y_{G|E}^* = \frac{1 + m_E}{1 + m_E + \mu} \left(R_G + V' - \frac{W'}{1 + m_E} \right), \quad (74)$$

while y_G^{**} is given by (42). Using (74) and (42), therefore, we can determine the optimal value for m_E , and consequently, the optimal value for k_E .

If this solution could be implemented, it would seem to be the best policy. The most important roadblock to the implementation of this policy is the concept of taking a flat tax from the industry for investment in Y . To the extent that government intends to return that tax to the industry through the matching subsidy, prudent member firms that recognize the free rider behavior of individual members should not object.

However, numerical computations with reasonable values for the parameters involved indicate that even if an industry accepts this plan in principle, the magnitude of the flat tax involved may often make them apprehensive. The Appendix shows that in some cases the theoretical value of k_E exceeds the total investable resources (R_G) available to the entire industry! Of course, the corresponding government matching rate is also very high (8.0497). But certainly, to implement this measure of intervention, government would have to convince the industry that in a sense the value of k_E is hypothetical and at the operating levels of resource allocations the matching subsidy truly nullifies the high magnitude of the flat tax.

In summary, *our analysis suggests a method for constructing instruments of governmental intervention that are optimal according to our measures of effectiveness and efficiency.*

5. Discussion and Conclusions

The analysis presented above represents only a small part of our work [7]. First, the model herein described is only one of those that we have formulated. Some of our models deal with an industry’s allocation of resources to applied research and development (i.e., appropriate but risky research activities), others deal with situations in which benefits from R&D activities and non-R&D activities are interdependent, and still others consider situations in which benefits are distributed not exponentially, but according to other probability distributions. Although these other models have shown some strikingly different results, we are encouraged by the fact that none of them have contradicted the primary findings concerning the effectiveness and efficiency of the provision of seed money or matching subsidies obtained from the model presented here. Hence, our models suggest that the federal government should avoid the use of seed money as a means of stimulating basic research in industry, for such expenditures in fact reduce the industry’s own investment. The models further suggest

that matching subsidies be available only beyond a minimum investment on the part of the industry, and that where possible our optimal intervention instrument (involving a tax return through a matching subsidy) be used.

Secondly, the model we have presented is, in spite of its many assumptions and consequent limitations, very versatile. It has been subjected to a variety of interesting analyses, some of which led to the following important recommendations for government policy towards the stimulation of basic research in industrial firms:

(a) Where feasible, government should attempt to increase industry's cooperative spirit and predisposition toward R & D investment (i.e., increase intra-industry matching rates). This can be done through interfirm communication, seminars, and measures similar to the creation of Cotton Inc.¹¹

(b) Reduce inter-firm competition (i.e., increase the value of α_{xi}) but increase inter-industry competition (i.e., reduce the values of α_{yi}).

(c) Increase intervention in industries involving greater number of firms. Alternatively, reduce the number of firms in an industry (at least by not subsidizing the smaller marginal firms).

(d) Tax the benefits from private investments (X_i) more heavily and those from joint investment less heavily.

(e) Do not pay special attention to the degree of heterogeneity of industries.

(f) Although industries involving: (i) smaller number of firms, (ii) greater investable resources, (iii) greater inter-industry competition, (iv) less interfirm competition and (v) greater risk aversion have relatively less need for government intervention, in most industries the degree of suboptimality of investment in inappropriate joint opportunity is fairly high. It would seem useful to employ the models and methods of analysis of this paper to determine degrees of suboptimality involved and appropriate government intervention methods.

Lastly, in this paper we have not presented any empirical evidence in support of the model or analysis. While we have obtained some evidence in support of some of the intermediate results of our models, we have found that conclusive evidence is difficult, if not impossible, to obtain without active government intervention designed to be experimental. Hopefully, governmental policy makers will want to carry out appropriate experimentation to test our recommendations.

Appendix. A Numerical Example

During the development of the models and analyses we worked through several numerical examples with the following purposes in mind: (i) to ensure the algebraic accuracy of the models; (ii) to observe the pattern of relationships among different variables, particularly when algebraic manipulation did not provide conclusive results; and (iii) to obtain a "feel" for the relative magnitudes of investment under alternative sets of values for the parameters.

In the table, we present one numerical example that facilitates a better understanding of some of our findings in this paper. This example illustrates the effectiveness and efficiency of alternative instruments of government intervention.

¹¹In 1968, Congress created Cotton Inc. to pursue cooperative R & D for cotton producers. Cotton Inc. was given a taxing authority to collect the costs of its R & D programs in proportion to individual cotton producers' tonnage of production. However, a voluntary feature was incorporated. Individual producers could apply in writing to get back their contributions. Because amounts involved were small (an average of \$2.50 per producer per month, to be applied for return on a monthly basis), only a few producers actually applied for the return of their contributions. Thus, in spite of the voluntary feature in this case, the values of m_{G-i} involved were higher than they would have been in the absence of external intervention. However, in a general case, we would not recommend the inclusion of the voluntary feature.

An Example of the Effect of Various Values of Seed Money and Matching Subsidy Rates on the Industry's Investment in Basic Research

Pair #	k_E	m_E	$y_{i E}^*$	$S_{i E}$	$N_{i E}$	y_E	e	y_T^*	S_T
1	0	0	503.589	0.842	0	0	—	5,035.89	0.842
2	1,000	0	413.499	0.870	-90.090	1,000.000	-0.901	5,134.99	0.839
3	5,000	0	53.139	0.983	-450.450	5,000.000	-0.901	5,531.39	0.826
4	0	0.1	516.667	0.838	13.078	516.667	0.253	5,683.34	0.822
5	0	0.5	551.542	0.827	47.953	2,757.710	0.174	8,273.13	0.740
6	0	4.144	619.481	0.805	115.892	25,671.305	0.045	31,866.12	0.000
7	-1,000	0.	593.679	0.814	90.090	-1,000.000	-0.901	4,936.79	0.845
8	-29,777	0.	3,186.830	0.000	2,683.241	-29,777.000	-0.901	2,091.30	0.934
9	-256,530	8.0497	3,186.83	0.000	2,683.241	0	∞	31,868.83	0.000

Note: Most symbols here have been defined in the text. The following new ones may be added.

$S_{i|E}$ = Suboptimality of each firm's (and industry's) own investment = $(y_i^{**} - y_{i|E}^*)/y_i^{**}$,

S_T = Suboptimality of total (including government's) investment = $(y_G^{**} - y_T^*)/y_G^{**}$,

$N_{i|E}$ = The net increase in a firm's investment in Y because of government intervention = $y_{i|E}^* - y^*$.

We consider a homogeneous group of firms in an industry with:

$$\begin{array}{lll} \alpha_X = 20 & \alpha_Y = 10 & r = 0.0006 \\ \beta_X = 1.1 & \beta_Y = 1.05 & R = 5000 \\ & m = 0.1 & g = 10 \end{array}$$

Note that subscript i is simply dropped from all variables since all 10 firms have the same values for the various parameters. Given these numerical values for parameters, in the absence of government intervention [$k_E = 0$, $m_E = 0$], $y^* = 503.589$ and $y^{**} = 3186.83$. Consequently, the degree of suboptimality $S = (y^{**} - y^*)/y^{**} = 0.842$.

We consider several pairs of values of k_E and m_E and examine their joint effect.

That seed money (k_E) is counter-productive is illustrated by the first three rows which consider positive values of k_E . That a matching subsidy is effective but considerably costly for the government is illustrated by the next three rows. Rows 7 and 8 illustrate the impact of negative values for k_E . That is, they consider the possibility of a flat tax. The last row presents the "optimal" pair of values of k_E and m_E in accordance with our proposal.

Probably the most important thing to note here is the magnitude of the flat tax (\$256,530) under the optimal instrument in comparison to the total investable resources of the industry (\$50,000). Of course, since the corresponding government matching is also very high, for all practical purposes the initial flat tax is only hypothetical.

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