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A HOMOGENEOUS INDUSTRY MODEL OF RESOURCE ALLOCATION TO BASIC RESEARCH AND ITS POLICY IMPLICATIONS*

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In a recent paper we showed that unaided industry allocation to basic (inappropriable) research is suboptimal and that in stimulating this allocation, provision of government seed money is generally counterproductive, while the provision of matching subsidies is not cost-efficient. Here we consider a special case of the model developed in the earlier paper (i.e. we now consider a homogeneous industry) and investigate the effects of several relevant factors upon an industry's allocation of resources to basic research. An extensive numerical example is presented that helps to verify and to interpret the model in realistic terms. Our findings question the validity of a number of popular beliefs about the need for government support of basic research in various types of industries. For example, contrary to popular belief, the greater the risk aversion displayed by member firms in an industry, the lesser may be the need for government support of its basic research. Also, the larger the number of firms in an industry the greater may be the need for government support of that industry's basic research.
(RESEARCH AND DEVELOPMENT; ECONOMICS; GOVERNMENT)

1. Introduction

Economists have long contended that a free market system will fail to attain optimal allocation of resources to research and development (R & D). Arrow (1962) pointed out two reasons for such an underinvestment: (i) the risk inherent in any R & D activity and (ii) the inappropriability of the benefits of many R & D activities.[1]

Not all R & D activities possess the above two characteristics to the same degree. For example, basic research possesses a high degree of inappropriability in addition to some risk, whereas applied research and development activities are likely to be largely appropriable (under U.S. patent laws) but highly risky. In any case, the existence of either one of the two characteristics is sufficient for the free market system to underinvest in an R & D activity.[2] On the other hand, the effects of risk and inappropriability on the degree of underinvestment may not be necessarily additive—particularly when interfirm cooperation is permitted.

Recently, we (Joglekar and Hamburg 1983a, b) published models of resource allocation behavior of a group of firms considering different types of R & D activities. Our analysis showed that unaided industry allocation to basic (inappropriable) research is suboptimal, but if firms are permitted to share the costs and benefits of applied research and development (assumed to be characterized by risk and appropriability), in most cases, such activities may be funded. We used our models to evaluate the effectiveness and efficiency of alternative government policies aimed at stimulating different types of R & D activities. We showed that in stimulating industry allocation to

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[1] For a detailed discussion of Arrow's argument and some counterarguments see the text and notes in Joglekar and Hamburg (1983a).

[2] Strictly speaking, it is not right to label all basic research as inappropriable or all applied research as appropriable. But for a further discussion of this point and an understanding of our use of these terms see Joglekar and Hamburg (1983a).

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basic research, provision of government seed money is generally counterproductive, while the provision of matching subsidies is not cost-efficient (Joglekar and Hamburg 1983a). We also pointed out that current federal policies toward interfirm cooperation often deprive applied research and development activities of their appropriability. Consequently, these policies encourage underinvestment in R&D instead of correcting it (Joglekar and Hamburg 1983b).

Our models (or their special cases) have many more implications for government policy that could not be discussed in the previous publications. In this paper, we try to elaborate on some of the additional policy implications. In §2, we recapitulate a special case of our model in Joglekar and Hamburg (1983a). Manipulating that model further, §3 analyzes several industry characteristics that influence the degree of suboptimality of industry allocation to basic (risky and inappropriable) research. §4 provides a numerical example which illustrates and confirms the theoretical analysis of §3. §5 discusses some of the policy implications of this analysis as well as our earlier analysis for policymakers in government.

2. The Model

Here we consider a special case of the model presented in an earlier paper (Joglekar and Hamburg 1983a). Let us restrict ourselves to a homogeneous industry (all firms identical) consisting of g firms and acting in the absence of any government intervention. Briefly, we can recapitulate the notation and the model as below:

$G \equiv \{1, 2, \ldots, i, \ldots, g\}$ a group of g firms,

X_i = An appropriable investment opportunity for firm i,

x_i = i's investment in X_i.

Firm i's benefits from X_i are assumed to be exponentially distributed[3] with the mean $(\alpha_X + \beta_X x_i)$, where

α_X = Index of general competitive advantage of firm i and

β_X = the increment in mean benefit per dollar of the firm's investment in X_i.

Y = Basic research investment opportunity whose benefits are available to each firm in G regardless of the amounts invested by each firm.

y_i = i's investment in Y.

Firm i perceives that other firms together will contribute a fixed sum k to Y regardless of what i does and will match every dollar of y_i with m dollars. (In a homogeneous industry $0 \leqslant m \leqslant g - 1$.)

Therefore, y_G, the group's total investment in Y is given by

$$y_G = k + (1 + m) y_i . \tag{1}$$

Note that the larger the value of m the greater is the cooperative spirit in the industry. In general, we expect m to be close to zero.

Firm i's benefits from Y are exponentially distributed[3] with the mean $(\alpha_Y + \beta_Y y_G)$, where,

α_Y = a priori advantage of each firm because of past basic research in the industry, and

β_Y = increase in mean benefits of each firm per dollar of total industry investment in Y.

We assume that each firm has an exponential utility function[4] with r as the risk aversion factor ($r \geqslant 0$). Furthermore, each firm has R dollars to allocate between X_i and Y.

[3]For the rationale underlying the assumed exponential distribution see footnote 5 of Joglekar and Hamburg (1983a).

[4]For the rationale underlying the assumption of exponential utility, see the text in Joglekar and Hamburg (1983a).

Under these assumptions, by equation (32) of our earlier paper (Joglekar and Hamburg 1983a), the industry's individually rational equilibrium allocation to Y is given by

$$y_G^* = \frac{(1+m)g}{g+1+m}\left(R + V - \frac{W}{1+m}\right) \quad \text{where} \tag{2}$$

$$V = (1 + r\alpha_X)/(r\beta_X), \tag{3}$$

$$W = (1 + r\alpha_Y)/(r\beta_Y). \tag{4}$$

Similarly, by equation (42) of the earlier paper (Joglekar and Hamburg 1983a), the Pareto optimal[5] allocation is given by

$$y_G^{**} = \tfrac{1}{2}(R + V - W/g). \tag{5}$$

Note that since $0 \leqslant m \leqslant g - 1$,

$$y_G^* \leqslant y_G^{**}, \tag{6}$$

and, in general, since m is close to zero, y_G^* is considerably short of y_G^{**}. The shortfall of y_G^* from y_G^{**} is exactly what we call an underinvestment in basic research. Let us define the degree of suboptimality, S, as

$$S = \frac{y_G^{**} - y_G^*}{y_G^{**}}.$$

Then, from (2) and (5), we can show that

$$S = \frac{(g - m - 1)}{(g + m + 1)} \cdot \frac{(gR + gV + W)}{(gR + gV - W)}. \tag{7}$$

With the degree of suboptimality so defined, we can now analyze the sensitivity of this degree of suboptimality to each of the different variables involved in the model. This sensitivity analysis will help us identify situations in which the need for government intervention[6] is relatively greater, assuming that government wants to minimize the degree of suboptimality.

[5]The concept of Pareto optimality is discussed more fully in Joglekar and Hamburg (1983a) and Joglekar (1978).

[6]By "government intervention" we do not necessarily mean "government funding", although that is a possible option for the government. In our concept, a higher degree of government intervention could have one or more of a variety of manifestations including:

(a) Relaxing the pursuit of technical violations of the antitrust laws in the industry under consideration,

(b) Legislating the creation of a cooperative R & D organization with (or without) "taxing" authority,

(c) Creating a climate and culture for cooperation in basic research,

(d) Encouraging international transfer of technology in the targeted industry,

(e) Providing seed money for the initial R&D or administrative costs of an industry-wide cooperative venture for basic research,

(f) Providing matching subsidies, either for individual firms, or for the industry's cooperative venture, for specific types of basic research activities,

(g) Underwriting the education and training costs of scientists most likely to be employed in the industry of concern,

(h) Funneling substantial amounts of money to university research directed at the concerned industry's products and/or processes,

(i) Legislating "acceptable standards" of performance for the industry's products and/or processes by a future date.

(j) Granting contracts for basic research relevant to the industry,

(k) Setting up a government laboratory to do the relevant basic research, etc.

In short, the way we use the term "government intervention" should not initiate the classical discussion on the *efficiency* of private versus government funding of basic research.

3. The Analysis

3.1. *Sensitivity to the General Competitive Advantage of a Firm*

We have defined α_X as the index of general competitive advantage of a firm in the industry. Differentiating S with respect to (w.r.t.) α_X, we obtain

$$\frac{\partial S}{\partial \alpha_X} = \frac{\partial S}{\partial V} \cdot \frac{\partial V}{\partial \alpha_X} = \frac{(g-m-1)}{(g+m+1)} \cdot \frac{\left(-\frac{2g}{\beta_X} \cdot W\right)}{(gR+gV-W)^2}. \tag{8}$$

It is easy to verify that under our assumptions $\partial S/\partial \alpha_X$ will be negative. *Thus, other things equal, the greater the general competitive advantage an average firm in the industry has, the smaller would be the suboptimality of group investment in Y, and hence the smaller would be the need for government intervention.*

Note that α_{Xi} is the benefit that firm i obtains even when it decides to invest no new capital in its appropriable investment opportunity. The value of α_{Xi} depends upon such things as a firm's patent position, current capacity, market share, market niche, etc. In a homogeneous industry, by definition, all α_{Xi} are equal ($= \alpha_X$). Still, in industries where interfirm competition is relatively less, e.g., due to regionally segmented monopolies in the market, α_{Xi} is likely to be higher than in industries with intense interfirm competition. Our results says that *an industry that manifests relatively more intense interfirm competition will operate at a higher degree of suboptimality of investment in basic research, and would have a higher need for government intervention.*

3.2. *Sensitivity to Expected Benefits per Dollar of Investment in the Individual Opportunity X*

α_Y represents the a priori advantage each firm in the industry enjoys because of past basic research of the industry as a whole. Differentiating S w.r.t. α_Y we obtain

$$\frac{\partial S}{\partial \alpha_Y} = \frac{\partial S}{\partial W} \cdot \frac{\partial W}{\partial \alpha_Y} = \frac{(g-m-1)}{(g+m+1)} \cdot \frac{\left[\frac{2}{\beta_Y}(gR+gV)\right]}{[gR+gV-W]^2}. \tag{9}$$

Under our assumptions $\partial S/\partial \alpha_Y$ will be positive. Thus, other things being equal, *the greater the general competitive advantage of an industry over other industries, the greater will be the degree of suboptimality of investment in Y.*

Note that α_Y is the benefit each firm in the industry obtains even when the industry as a whole invests no new capital in its basic research. In addition to the industry's past investment in its basic research, a number of other factors can influence the value of α_Y. For example, if an industry's product(s) has little competition from substitute products, its α_Y would be larger than that for an industry with a serious threat from substitute products. Our result says that *an industry with little competiton from substitute products will fall farther short of its group rational investment in Y (its basic research) in comparison to an industry that is faced with difficult competition from substitute products.* The same logic applies to an industry faced with intense competition from imports. In the past, industry groups faced with such intense competition have been successful in persuading the government specifically to encourage their cooperative basic research efforts. Our result suggests that such special encouragement to these industries is not necessarily justified insofar as industries that are faced with relatively less serious competition are likely to be farther short of the Pareto optimal investment in their basic research.

Similarly, another factor influencing the value of α_Y is the intensity of government regulation affecting an industry. For example, an industry that is faced with strict legislative standards for safety and environmental impact of its products would display a smaller value of α_Y than an industry faced with little regulation. Our result says that *an industry group faced with little regulation will fall farther short of its optimal investment in basic research than an industry group that is faced with strict regulation.* Thus, our result suggests that in the past government may have given special attention to basic research in the wrong industries.

Of course, it should not be forgotten that investment in basic research is suboptimal in all kinds of industry. Here, we have been focusing only on *the relative magnitude* of that suboptimality in different types of industries.

3.3. *Sensitivity to Expected Benefits per Dollar of Investment in the Individual Opportunity X*

In the homogeneous case, β_X is the measure of expected benefits per dollar of investment in the individual opportunity. Differentiating S w.r.t. β_X we have

$$\frac{\partial S}{\partial \beta_X} = \frac{\partial S}{\partial W} \cdot \frac{\partial W}{\partial \beta_X} = \frac{(g-m-1)}{(g+m+1)} \cdot \frac{\left(\frac{2gr}{\beta_X} \cdot V \cdot W\right)}{[gR + gV - W]^2} \tag{10}$$

which is positive under our assumptions. Thus, other things being equal, the larger the expected benefits per dollar of investment in X, the greater is the degree of suboptimality of investment in Y, a result that makes intuitive sense.

3.4. *Sensitivity to Expected Benefits per Dollar of Investment in the Joint Opportunity Y*

β_Y is the measure of expected benefits per dollar of investment in the joint opportunity Y. Differentiating S w.r.t. β_Y we have

$$\frac{\partial S}{\partial \beta_Y} = \frac{\partial S}{\partial W} \cdot \frac{\partial W}{\partial \beta_Y} = \frac{(g-m-1)}{(g+m+1)} \cdot \frac{\left[-\frac{2rW}{\beta_Y}(gR + gV)\right]}{[gR + gV - W]^2} \tag{11}$$

which is negative. Thus we have another result that makes intuitive sense, namely that, *other things being equal, the larger the expected benefits per dollar of investment in Y, the smaller is the degree of suboptimality of investment in Y.*

3.5. *Sensitivity to Investable Resources of Each Firm*

Differentiating S w.r.t. R, we have

$$\frac{\partial S}{\partial R} = \frac{(g-m-1)}{(g+m+1)} \cdot \frac{[-2gW]}{(gR + gV - W)^2} \tag{12}$$

which is clearly negative under our assumption. Thus, *other things being equal, the larger the investable resources of the industry the lesser is the degree of suboptimality of investment in Y.*

3.6. *Sensitivity to Cooperative/Exploitative Climate in an Industry*

Differentiating S w.r.t. m we have

$$\frac{\partial S}{\partial m} = \frac{-2g}{(g+m+1)^2} \cdot \frac{(gR + gV + W)}{(gR + gV - W)} \tag{13}$$

which is negative under our assumption.

Thus, other things being equal, the greater the value of the perceived matching rate (m) *the lesser will be the degree of suboptimality of group investment in Y.* We have said that the value of m reflects the cooperative/exploitative climate in an industry. Our result says that the greater the perceived cooperation (which must be actual in equilibrium), the lesser will be the degree of suboptimality. Alternatively, the greater the interfirm rivalry (and perceived exploitation) the greater will be the suboptimality.

Note further that the value of m depends on whether or not a group is organized. In organized groups m may be larger, and hence the degree of suboptimality smaller. Of course, we must note that even in organized groups unless there is coercion (i.e., the organization has taxing power) and complete knowledge, the true value of m may be quite small in relation to g because of the free rider problem.

In unorganized (or voluntary) groups, m may fall short of g. Particularly in large voluntary groups, a firm is likely to believe that its own actions have no effect whatsoever on the resource allocations of the remaining firms. In economics, this belief is referred to as the Cournot assumption and implies that $m = 0$.

Note that when the Cournot assumption is valid,

$$S > \frac{g-1}{g+1}. \tag{14}$$

Thus, even a group consisting of two identical firms will display a degree of suboptimality greater than 0.33. A homogeneous group consisting of 10 firms will fall short of its group rational investment by at least 80%.

For any student concerned with the need for government intervention this significant magnitude of suboptimality is worth noting. It is evident that in relatively homogeneous industries consisting of large numbers of firms (where the Cournot assumption is particularly valid) government intervention[6] is required. We shall discuss the sensitivity of S to g more formally in a later section.

3.7. *Sensitivity to Risk Aversion on the Part of Each Firm*

Differentiating the S w.r.t. r we have

$$\frac{\partial S}{\partial r} = \frac{\partial S}{\partial V} \cdot \frac{\partial V}{\partial r} + \frac{\partial S}{\partial W} \cdot \frac{\partial W}{\partial r}, \tag{15}$$

$$\frac{\partial S}{\partial r} = \frac{(g-m-1)}{(g+m+1)} \cdot \frac{\frac{2g}{r^2} \cdot \left(\frac{W}{\beta_X} - \frac{R+V}{\beta_Y} \right)}{(gR + gV - W)^2}. \tag{16}$$

The sign of this expression depends upon the sign of

$$\left(\frac{W}{\beta_X} - \frac{R+V}{\beta_Y} \right) \tag{17}$$

i.e., on the sign of

$$\left[\alpha_Y - (\alpha_X + \beta_X R) \right]. \tag{18}$$

Note that α_Y is the mean and the standard deviation of each firm's benefits from the industry's prior basic research, whereas $(\alpha_X + \beta_X R)$ is the mean and the standard deviation of a firm's benefits when all of its resources are invested in its appropriable investment opportunity.

It would indeed be a rare industry where α_Y would be greater than $(\alpha_X + \beta_X R)$. Thus, in general, we expect the sign of $\partial S / \partial r$ to be negative.

Thus, the greater the degree of risk aversion displayed by the member firms in industry, the lesser is the suboptimality of the industry's investment in basic research.

This finding seems to contradict what we had inferred from Arrow's (1962) work. Although Arrow did not analyze relative effects of different levels of risk aversion, based on his discussion we had hypothesized that industries displaying greater risk aversion would invest more suboptimally in their basic research than industries displaying relatively smaller risk aversion.[7] We believe that this hypothesis has been widely held by a number of students of government policy concerning R & D.

Our analysis shows that this popularly held hypothesis is incorrect. Instead, we find that greater risk aversion leads to a smaller degree of suboptimality of investment in basic research.

Of course, Arrow's (1962) major focus was to identify the variety of factors that could lead to suboptimal investment in R & D on the part of a private enterprise system. The risk associated with R & D activities was one of these factors. Implicit in Arrow's discussion (which considered only the allocation to R & D, and not allocation of resources between R & D and other investment opportunities) was the assumption that investment opportunities other than R & D had relatively low risks associated with them. Our model is clearly different. We assume that both the basic research activity and the alternative investment opportunity have specific risks associated with them. In our model, we find that if the mean (and the standard deviation) of a firm's benefits from its industry's prior basic research is smaller than the mean (and the standard deviation) of its benefits when all of the firm's own resources are invested in the alternative appropriable investment opportunity (a very likely case), lesser risk aversion leads to a greater degree of suboptimality of investment in basic research.

In any case, Arrow's statement that "we expect a free enterprise economy to underinvest in invention and research (as compared with an ideal) because it is risky, because the product can be appropriated only to a limited extent and because of increasing returns in use," (p. 619) perhaps leaves the impression that these factors are additive as far as their effect on suboptimality of investment in R & D is concerned.

Our analysis takes into account the *joint effect of inappropriability of an R & D activity and risk aversion on the part of individual firms* and suggests that if an industry is concerned with a basic research investment opportunity, risk aversion on the part of member firms may in fact help the industry in reducing the degree of suboptimality of its investment in the basic research.[8] This demonstration, in itself, is an important contribution of this paper to the pertinent literature.

In short, we can say that if two industrial groups are faced with their own basic research investment opportunities, other things being equal, the group whose member firms are relatively more risk averse may invest less suboptimally in its basic research than a group whose members are less risk averse. Consequently, in contradiction to the popular belief, *a risk averse industry calls for less government intervention*[6] *than an industry with lesser risk aversion.*

[7] Perhaps we were led to this inference because Arrow had stated,

> . . . any unwillingness or inability to bear risks will give rise to nonoptimal allocation of resources, in that there will be discrimination against risky enterprises as compared with optimum. A preference for risk might give rise to misallocation in the opposite direction, ... (Arrow 1962, pp. 611–612).

On the other hand, perhaps we had arrived at our hypothesis purely intuitively and independent of Arrow's work.

[8] Perhaps one reason why we have obtain this result is that we have used Pareto optimality (which accepts individual risk aversion, as is) as a surrogate for social optimality. We do not know what result would be obtained if social optimality is based on the concept that society (or an industry group as a whole) ought to be risk neutral. Our model is not suitable to investigate that type of situation, and that inquiry must be left for the future. However, we mention this possibility, because Arrow's discussion implies risk neutrality on the part of the society. For example, he says "On the other hand, (from society's point of view,) such activities should be undertaken if expected return exceeds the market rate of return, no matter what the variance is." (Parenthesis added by us, Arrow 1962, p. 613).

3.8. *Sensitivity to Group Size in a Homogeneous Industry*

The study of sensitivity to group size is more complex than the study of sensitivity to other variables in our model. This is because several other variables may be dependent on group size. For example, if we want to study the effect of group size holding total investable resources of the group constant, the resources available to each firm will be a decreasing function of g. That is, $R = R_G/g$. Elsewhere, (Joglekar 1978), we have studied the effect of group size under a variety of circumstances. Here we consider only the simple case when R, V, W, m are independent of g.

Then, differentiating S w.r.t g we get

$$\frac{\partial S}{\partial g} = \frac{2(m+1)\left[g^2(R+V)^2 - W^2\right] - 2W(R+V)\left[g^2-(m+1)^2\right]}{\left[(g+m+1)(gR+gV-W)\right]^2}. \tag{19}$$

The sign of $\partial S/\partial g$ depends upon the sign of the numerator (NUM19) of equation (19).

Furthermore, when Y_G^* in (2) is positive (i.e., when the solution is a noncorner solution) we know that

$$W < (1+m)(R+V). \tag{20}$$

Hence we know that

$$\text{NUM19} > 2(m+1)(R+V)^2\left[g^2-(1+m)^2\right] - 2(m+1)(R+V)^2\left[g^2-(1+m)^2\right], \tag{21}$$

i.e., NUM19 > 0. Hence, $\partial S/\partial g > 0$.

Thus, in this case, as group size increases, the degree of suboptimality of investment in basic research increases.

Elsewhere (Joglekar 1978), we have considered other cases of changes in group size and found that in each case, as group size increases, the degree of suboptimality of investment in Y increases. Thus, in general, *an industry consisting of a larger number of firms has a greater need for government intervention*[6] than an industry consisting of a small number of firms.

At this point we would also like to remind the reader of the earlier discussion in §3.6 where we pointed out that if the Cournot assumption is valid, the need for government intervention is quite intense even in a group as small as 10 or 15 firms.

Thus, in this paper we have analyzed the effects of a number of industry characteristics on the suboptimality of the industry's investment in basic research. §4 presents a numerical example. Then, in §5, we turn to a more comprehensive discussion of the policy implications of our models based on the analysis presented in this paper as well as the analysis presented in our earlier papers (Joglekar and Hamburg 1983a, b).

4. A Numerical Example

During the development of the models and analyses we worked through several numerical examples with the following purposes in mind: (i) to ensure the algebraic accuracy of the models; (ii) to observe the pattern of relationships among different variables, particularly when algebraic manipulation does not provide conclusive results; and (iii) to obtain a "feel" for the relative magnitudes of investment under alternative sets of values for the parameters.

In Table 1, we present a numerical example that facilitates a better understanding of some of the findings in this paper. This example illustrates the sensitivity of the degree of suboptimality of an industry's investment in its basic research to the various parameters in our model.

TABLE 1

Numerical Example

No.	Assumptions								Results				
Run No.	α_X ($million)	β_X	α_Y ($million)	β_Y	m	r $\times 10^{-7}$	R ($million)	g	V ($million)	W ($million)	y_G^* ($million)	y_G^{**} ($million)	S
1	1.00	0.75	0.50	0.10	4	2	5	10	8.000	55.000	6.667	37.500	0.822
2	2.00	0.75	0.50	0.10	4	2	5	10	9.333	55.000	11.111	44.167	0.748
3	1.00	0.85	0.50	0.10	4	2	5	10	7.059	55.000	3.529	32.794	0.892
4	1.00	1.00	0.50	0.10	4	2	5	10	6.000	55.000	0.000	27.500	1.000
5	1.00	1.10	0.50	0.10	4	2	5	10	5.455	55.000	− 1.818	24.773	1.073†
											(0.000)		(1.000)
6	1.00	0.40	0.50	0.10	4	2	5	10	15.000	55.000	30.000	72.500	0.586†
												(50.000)	(0.400)
7	1.00	0.75	0.80	0.10	4	2	5	10	8.000	58.000	4.667	36.000	0.870
8	1.00	0.75	0.50	0.15	4	2	5	10	8.000	36.667	18.889	46.667	0.595
9	1.00	0.75	0.50	0.10	6	2	5	10	8.000	55.000	21.176	37.500	0.435
10	1.00	0.75	0.50	0.10	9	2	5	10	8.000	55.000	37.500	37.500	0.000
11	1.00	0.75	0.50	0.10	0	2	5	10	8.000	55.000	− 38.182	37.500	2.018†
											(0.000)		(1.000)
12	1.00	0.75	0.50	0.10	3	2	5	10	8.000	55.000	− 2.143	37.500	1.057†
													(1.000)
13	1.00	0.75	0.50	0.10	4	4	5	10	4.667	30.000	12.222	33.333	0.633
14	1.00	0.75	0.50	0.10	4	1	5	10			− 4.444	45.833	1.097†
											(0.000)		(1.000)
15	1.00	0.75	0.50	0.10	4	2	7	10	8.000	55.000	13.333	47.500	0.719
16	1.00	0.75	0.50	0.10	4	2	3	10	8.000	55.000	0.000	27.500	1.000
17	1.00	0.75	0.50	0.10	4	2	5	12	8.000	55.000	7.059	50.500	0.860
18	1.00	0.75	0.50	0.10	4	2	5	6	8.000	55.000	5.455	11.500	0.526
19	1.00	0.75	0.50	0.10	4	2	5	20	8.000	55.000	8.000	102.500	0.922†
												(100.000)	(0.920)
20	1.00	0.75	0.50	0.10	4	2	5	200	8.000	55.000	9.756	1272.500	0.992†
												(1000.000)	(0.976)

†Implies a "corner point solution." That is, the calculated values cannot be implemented. Actual values are shown in the brackets below.

Consider a group of ten firms ($g = 10$) each with \$5 million available for long-term investment ($R =$ \$5 million), to be allocated between two types of opportunities X and Y. X represents an appropriable opportunity (e.g., plant investment, patentable R & D, etc). Each firm's benefits B_X from an investment x in X are assumed to be exponentially distributed with the mean \$(1,000,000 + 0.75x). In other words, even if a firm invests nothing at all in X, because of its historical competitive position in the industry, it would obtain certain benefits in the future, the expected net present value of which is \$1,000,000. In addition, for every dollar it spends on X, the firm would obtain certain *net* benefits (revenue minus cost) in the future with an expected present value of \$0.75. In short we assume $\alpha_X =$ \$1,000,000 and $\beta_X = 0.75$.

Y represents an inappropriable opportunity (e.g., basic research). Each firm stands to benefit from the total investment y_T in Y regardless of which firm contributed what portion of that total investment. Let each firm's benefits B_Y from y_T be distributed exponentially with the mean (\$500,000 + 0.10y_T). In other words, even if all the firms in this industry together spend nothing at all on their basic research, because of the industry's historic competitive position (vis-a-vis other industries), each firm will obtain net benefits with expected present value of \$500,000. In addition, for every additional dollar spent by the industry on its basic research, each firm obtains a net benefit with expected present value of \$0.10. That is, $\alpha_Y =$ \$500,000 and $\beta_Y =$ \$0.10. Assume further that each firm believes that every dollar it invests in Y will be matched by the rest of the industry (all other firms together) with \$4. That is $m = 4$. Lastly, we assume that each firm has an exponential utility function with the risk aversion factor, $r = 0.0000002$ (i.e. 2×10^{-7}).

With these numerical values, the individually rational equilibrium allocation to its basic research by the industry as a whole, as given by equation (2), is $Y_G^* =$ \$6.667 million. On the other hand, the industry's Pareto-optimal allocation as given by equation (5) is $Y_G^{**} =$ \$37.5 million. Thus the actual allocation is 82.2% short of the Pareto optimal allocation, and we say that, S, the degree of suboptimality is 0.822. These numbers are presented as the base case (Run No. 1) in the attached table. The table also presents relevant values for V and W (as defined in equations (3) and (4)) for the reader's convenience.

Runs No. 2 to No. 20 show the effects of specific alternative values of the various parameters on the degree of suboptimality.

Run No. 2 shows that when α_X, competitive advantage of each firm due to its historic investment in appropriate opportunities (e.g. a modern plant), is \$2 million, the industry as a whole is willing to invest almost twice as much in its basic research as when α_x was only \$1 million. Of course, that doubling of investment in basic research cuts the degree of suboptimality only slightly from 0.822 to 0.748 because the degree of suboptimality is very large (perhaps quite realistically) in our base case. Furthermore, as α_x increases, both Y_G^* and Y_G^{**} increase. Y_G^* increases proportionately more; hence, the reduction in S. In any case, Run No. 2 confirms the analysis in §3.1. Similarly, Run No. 7 confirms the analysis of §3.2. It shows that other things being equal, the greater the general competitive advantage of an industry over other industries (α_Y), the greater is the degree of suboptimality of investment in basic research.

Runs No. 3 to No. 6 examine the sensitivity of S with respect to β_X. Confirming the analysis of §3.3, they show that as β_X increases so does the degree of suboptimality of investment in basic research. It can be seen that when $\beta_X \geqslant g\beta_Y$, there will be no investment in Y, although Pareto-optimally there should be some. Run No. 5 illustrates a "corner point solution". If β_X is 1.10 while other parameters remain unchanged from the base case, the calculated value of Y_G^* is negative. Of course, in this case the actual group investment in Y will be 0. In fact, strictly speaking, any time calculated Y_G^* is less than the threshold level π (even if $Y_G^* > 0$), the actual group investment in Y will be 0. See Joglekar and Hamburg (1983a). Run No. 8 confirms the results in §3.4.

Runs No. 9 to No. 12 test the sensitivity of S w.r.t. the perceived matching rate m. As can be seen S is a decreasing function of m and becomes zero at $m = g - 1$. Runs No. 11 and No. 12 illustrate two corner-point solutions.

Runs No. 13 and No. 14 test the sensitivity of S w.r.t. the risk aversion constant r. They confirm the findings of §3.7, namely, the greater the risk aversion the lesser is the degree of suboptimality of investment in basic research. As discussed in §3.7 this is contrary to conventional wisdom. It may be noted here that the effect of an increase in r is twofold. It decreases Y_G^*, the individually rational industry investment in Y, and also decreases Y_G^{**}, the Pareto-optimal investment in Y. The net result is a substantial decrease in the degree of suboptimality S.

Runs No. 15 and No. 16 show that as investable resources available to each firm increase, the suboptimality of investment in basic research decreases, and vice-versa.

Runs No. 17 and No. 20 show that the degree of suboptimality of investment in basic research is an increasing function of group size. They show that by the time the group size is 20 or so, the degree of suboptimality is very high (0.92) and that at $G = 200$, calculated S is almost equal to 1. But of course, Runs No. 19 and No. 20 illustrate yet another type of "corner point solution". Here calculated values of Y_G^{**} exceed the group's total investable resources. Consequently, the actual values of Y_G^{**} must be equal to $g \cdot R$ only.

In short, the various runs in our numerical example not only help illustrate the model but also vividly point out "corner-point" situations.

5. Policy Implications for the Federal Government

Our models confirm that unaided industry allocation to basic (inappropriable) research is suboptimal even when firms are permitted to conduct basic research jointly (but voluntarily). The suboptimality of investment in basic research depends on several industry characteristics. Industries with one or more of the following characteristics are likely to have a higher degree of suboptimality of investment in basic research:

(a) Industries involving a large number of firms,
(b) Industries with intense interfirm competition,
(c) Industries with a lesser threat from other industries,
(d) Industries with smaller amounts of investable resources, and
(e) Industries with relatively less risk averse firms.

Clearly, industries displaying these characteristics deserve more government assistance in their basic research programs. At the same time, it should be realized that the typical fiscal measures used by the federal government to stimulate basic research are cost-inefficient, if not counterproductive. Our analysis (Joglekar and Hamburg 1983a) has shown that provision of seed money is generally counterproductive, while the provision of a matching subsidy is not cost-efficient in increasing industry's allocation to basic research.

The case of applied (appropriable) R&D, on the other hand, is quite different (Joglekar and Hamburg 1983b). Federal government may be able to see that most of the socially desirable applied R&D activities are voluntarily funded by the firms in an industry, either individually or through joint projects with other firms, provided government policy does not deprive these activities of their appropriable nature. But industry characteristics that increase the degree of suboptimality of investment in applied R&D are quite different from those that increase the degree of suboptimality of investment in basic research. As far as applied R&D is concerned, the larger the number of firms in an industry the smaller is the degree of suboptimality (Joglekar and Hamburg 1983b). We have also shown that heterogeneous industries (consisting of some small firms and some large firms) are more likely to fund their applied R&D activities than homogeneous industries are (Joglekar and Hamburg 1983b).

In short, the most important implications of our research for government policy makers, as presented in this paper as well as in our other works cited, can be summarized as below:

> POLICY MAKERS MUST STOP THINKING THAT ALL R&D ACTIVITIES ARE ALIKE AND REQUIRE A SINGLE TYPE OF GOVERNMENT INTERVENTION. ALTHOUGH RISK AND INAPPROPRIABILITY ARE THE TWO MAJOR REASONS WHY THERE IS UNDERINVESTMENT IN R&D ACTIVITIES, THE EFFECTS OF RISK AND INAPPROPRIABILITY ON THE DEGREE OF SUBOPTIMALITY OF INVESTMENT ARE NOT NECESSARILY ADDITIVE. INDUSTRIES THAT ARE LIKELY TO UNDERINVEST IN BASIC (INAPPROPRIABLE AND RISKY) RESEARCH ARE QUITE DIFFERENT FORM INDUSTRIES THAT ARE LIKELY TO UNDERINVEST IN APPLIED (APPROPRIABLE AND RISKY) R&D. OUR ANALYSIS HAS SHOWN THAT GOVERNMENT POLICIES APPROPRIATE FOR STIMULATING BASIC RESEARCH ARE QUITE DIFFERENT FROM THOSE APPROPRIATE FOR STIMULATING APPLIED R&D.[9]

[9]The authors gratefully acknowledge very insightful comments by Professors Kenneth J. Arrow, Yale M. Braunstein and Mancur Olson.

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