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INDUSTRY INVESTMENT IN BASIC RESEARCH ASSUMING INTERDEPENDENCE OF BENEFITS FROM APPROPRIABLE AND INAPPROPRIABLE ACTIVITIES*

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Using a model of the resource allocation behavior of a group of firms, Joglekar and Hamburg (1983) demonstrate that unaided industry allocation to basic research is suboptimal and that in stimulating this allocation, provision of government seed money is generally counterproductive, while the provision of matching subsidies is not cost-efficient. Using basically the same model, but focusing on the case of a homogeneous industry, Joglekar and Hamburg (1986) further identify several industry characteristics that increase the degree of suboptimality of investment in basic research, and consequently, the need for government intervention. Joglekar and Hamburg (1983), (1986) assume that a firm's benefits from its investment in appropriable activities are independent of its benefits from the industry's total investment in the pertinent basic (i.e., inappropriable) research. Investment theory suggests that a firm's benefits often depend upon its investment *portfolio* and that an investment in basic research that is not supported by suitable investments in other activities (e.g., plant and equipment, personnel training, etc.) may yield far smaller benefits than could be potentially obtained. Therefore, in this paper, I present a model that assumes such an interdependence of benefits. I find that most of Joglekar and Hamburg's (1983), (1986) conclusions are confirmed by the new model.
(INDIVIDUALLY RATIONAL INDUSTRY ALLOCATION; UNDERINVESTMENT; GOVERNMENT POLICY; ROBUST CONCLUSIONS)

1. Introduction

The failure of a free market system to attain socially optimal allocation of resources to research and development (R & D) is a generally recognized problem. Yet, we are just beginning to understand the types of R & D activities that receive relatively serious underinvestment, the types of industries that are more prone to underinvestment in certain kinds of R & D, and the types of government intervention strategies that are likely to be effective and efficient in the correction of that underinvestment in different types of industries. Using a model of the resource allocation behavior of a group of firms, Joglekar and Hamburg (1983) demonstrate that unaided industry allocation to basic inappropriable research is suboptimal and that in stimulating this allocation, provision of government seed money is generally counterproductive, while the provision of matching subsidies is not cost-efficient. Using basically the same model, but focusing on the case of a homogeneous industry, Joglekar and Hamburg (1986) explore several industry characteristics influencing the degree of an industry's underinvestment in basic research. For example, they find that industries with (i) greater investable resources, (ii) greater interfirm cooperation, (iii) greater risk aversion, and (iv) fewer firms display relatively less underinvestment in their basic research, and consequently need less government support.

While these findings are of great significance to government policy makers, their usefulness depends on the validity of the assumptions underlying Joglekar and Hamburg's (1983), (1986) models. Although most of their assumptions have been generally acceptable, scholars and referees have seriously questioned some of Joglekar and Hamburg's assumptions. One such assumption is that of *exponentially* distributed mean benefits

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from investments. Assuming *normally* distributed benefits, Joglekar and Hamburg (1987) find that most of their earlier conclusions remain valid.

Another one of Joglekar and Hamburg's (1983), (1986) assumptions that has been seriously questioned is that a firm's benefits from the appropriable and the inappropriable activities are independent of one another. Joglekar and Hamburg (1983), (1986) assume these benefits to be independent random variables, with the mean benefits from each type of activity being a function of only the investment in that type. Some scholars argue that *in-house basic research* allows a firm to absorb externally available knowledge. Others argue that basic research primarily increases the quality of a firm's downstream innovative activities. In that sense, its benefits lie in increasing the returns on the firm's future appropriable investments. Modern investment theory also suggests that a firm's benefits often depend upon its *investment portfolio*, and an investment in basic research that is not supported by complementary investments in other activities (e.g. plant and equipment, personnel training, advertising, etc.) may yield far smaller benefits than could be potentially obtained. Similarly, if a firm's plant investment is not adequately supported by its basic research, its benefits are likely to be far short of the potential.[1]

Therefore, it is important that Joglekar and Hamburg's (1983), (1986) findings and conclusions be verified by a model that assumes that a firm's benefits from its investments in appropriable activities are interdependent with its benefits from industry-wide investments in the related basic (inappropriable) research. This paper attempts to do precisely that. §2 describes the model which retains most of Joglekar and Hamburg's assumptions,[2] but introduces *interdependence of benefits* from the two types of investments. §3 uses this model to verify Joglekar and Hamburg's (1983) conclusions about the effectiveness and efficiency of government policies in stimulating basic research. §4 attempts to replicate Joglekar and Hamburg's (1986) analysis of the case of homogeneous industry. §5 concludes the paper with a discussion of the significance of our findings.

2. The Model

As in Joglekar and Hamburg (1983), let

$G \equiv (1, 2, \ldots, i, \ldots, g)$ an industry of g firms.

x_i = i's investment in X_i, an appropriable investment opportunity (e.g., plant investment, brand advertising).

y_i = i's investment in Y, the industry's basic research investment opportunity whose benefits are available to each firm in G, regardless of the amounts invested by each firm.

Firm i is assumed to perceive that other firms together will contribute a fixed sum k_{G-i} to Y regardless of what i does and will match every dollar of y_i with m_{G-i} dollars; $k_{G-i} \geq 0$, and $m_{G-i} \geq -1$. In the equilibrium, each firm's perception of other firms' behavior

[1] Freeman (1974) summarizes these arguments very well and also provides some empirical support for them. Rosenberg (1982, pp. 58–59) says, "The social payoff of an invention can rarely be identified in isolation. The growing productivity of industrial economies is the complex outcome of large numbers of interlocking, mutually reinforcing technologies, the individual components of which are of very limited economic consequence by themselves."

Finally, a referee for Joglekar and Hamburg (1989) said, "Basic research is in fact a complementary investment, being essentially a bridging activity between non-appropriated basic research skills and knowledge outside the firm, and downstream applied R & D, investment, and marketing. Benefits to the firm are therefore likely to be a positive function of

(a) the general stock of applicable basic research,
(b) the basic research undertaken by the firm itself and
(c) the firm's downstream innovative activities."

The referee suggested that this may be very difficult or impossible to model. I believe that this manuscript represents a decent attempt at such a model.

[2] For an enumeration and justification of those assumptions, see Joglekar and Hamburg (1983, pp. 997–1005) and Joglekar and Hamburg (1986, pp. 225–227).

is assumed to be actually borne out. Thus, y_G, the group's total investment in Y, is given by

$$y_G = k_{G-i} + (1 + m_{G-i})y_i. \tag{1}$$

Furthermore, if the government believes that, in the absence of its encouragement, industry's investment in basic research will be suboptimal, government may provide

$$y_E = k_E + m_E y_G \tag{2}$$

where k_E is the government seed money regardless of industry's investment in basic research, and m_E is the matching government subsidy per dollar of industry investment. We expect $k_E \geq 0$ and $m_E \geq 0$.

Given the foregoing assumptions, the total investment y_T in Y, where $y_T = y_G + y_E$, can be written as

$$y_T = k_E + (1 + m_E)k_{G-i} + (1 + m_E)(1 + m_{G-i})y_i. \tag{3}$$

Because Y is inappropriable, each firm's benefits depend upon this total investment y_T. We further assume that basic research is a lumpy good. That is, an investment in basic research would yield any meaningful results and economic benefits, only if $y_T \geqslant \pi$.

Now, unlike Joglekar and Hamburg (1983), we assume that a firm's benefits from its investment in X and from the total investment in Y are *interdependent* and their joint risk adjusted value (RAV) is given by

$$\left.\begin{aligned} \mathrm{RAV}_i &= (\alpha_{X_i} + x_i)^{\beta_{X_i}}(\alpha_{Y_i})^{\beta_{Y_i}} && \text{if} \quad y_T < \pi \\ \mathrm{RAV}_i &= (\alpha_{X_i} + x_i)^{\beta_{X_i}}(\alpha_{Y_i} + y_T)^{\beta_{Y_i}} && \text{if} \quad y_T \geq \pi \end{aligned}\right\} \tag{4}$$

where α_{X_i}, β_{X_i}, α_{Y_i} and β_{Y_i} are appropriate constants with values satisfying the following reasonable conditions:

$$\left.\begin{aligned} &\mathrm{RAV}_i > 0 \\ &\frac{\partial \mathrm{RAV}_i}{\partial x_i} > 0 \\ &\frac{\partial \mathrm{RAV}_i}{\partial y_T} > 0 \\ &\frac{\partial^2 \mathrm{RAV}_i}{\partial x_i^2} < 0 \\ &\frac{\partial^2 \mathrm{RAV}_i}{\partial y_T^2} < 0 \end{aligned}\right\} \quad \text{for} \quad x_i, y_T > 0. \tag{5}$$

Hence we assume that

$$\alpha_{X_i}, \alpha_{Y_i} > 0, \qquad 0 \leq \beta_{X_i}, \beta_{Y_i} \leq 1. \tag{6}$$

Notice that in order to keep the model simple we have not defined the benefit and the utility functions separately. Thus, with this model we will not be able to verify Joglekar and Hamburg's (1986) findings pertaining to risk aversion. Furthermore, this model does not allow as clear an interpretation of constants such as α_{X_i} and β_{X_i} as in Joglekar and Hamburg (1983). Nevertheless, we can say that, other things remaining the same, α_{X_i} reflects (but does not exactly measure) a firm's competitive advantage from its past investments in appropriable activities, and β_{X_i} reflects (but does not exactly measure) the marginal utility of a dollar invested in X_i. Similarly, α_{Y_i} reflects the firm's advantage from the industry's prior investment in basic research and β_{Y_i} reflects the marginal utility of a dollar invested in basic research.

Now, a rational firm would want to maximize its RAV subject to its resource constraint[3]

$$x_i + y_i = R_i \tag{7}$$

where R_i represents the investable resources at the disposal of firm i. If firm i expects y_T to be smaller than π, its optimal solution would be

$$x_i^* = R_i, \qquad y_i^* = 0. \tag{8}$$

This is the case of a "corner-point-solution".

When $y_T \geq \pi$, the optimal (noncorner) solution would satisfy (7), and the following equality

$$\frac{\partial \mathrm{RAV}_i}{\partial x_i} = \frac{\partial \mathrm{RAV}_i}{\partial y_i} \tag{9}$$

$$\text{that is,} \qquad \beta_{X_i}(\alpha_{Y_i} + y_T) = \beta_{Y_i}(\alpha_{X_i} + x_i)(\partial y_T/\partial y_i). \tag{10}$$

Using (3), (7), and (10) we can show that

$$\left.\begin{aligned} x_i^* &= \frac{\beta_{X_i}}{\beta_{X_i}+\beta_{Y_i}} R_i - \frac{\alpha_{X_i}\beta_{Y_i}}{\beta_{X_i}+\beta_{Y_i}} + \frac{\beta_{X_i}(\alpha_{Y_i} + k_E + (1+m_E)k_{G-i})}{(\beta_{X_i}+\beta_{Y_i})(1+m_E)(1+m_{G-i})}, \\ y_i^* &= \frac{\beta_{Y_i}}{\beta_{X_i}+\beta_{Y_i}} R_i + \frac{\alpha_{X_i}\beta_{Y_i}}{\beta_{X_i}+\beta_{Y_i}} - \frac{\beta_{X_i}(\alpha_{Y_i} + k_E + (1+m_E)k_{G-i})}{(\beta_{X_i}+\beta_{Y_i})(1+m_E)(1+m_{G-i})}. \end{aligned}\right\} \tag{11}$$

As Joglekar and Hamburg (1983) did, we assume that this individually rational solution describes the firm's actual behavior and that each firm's perception of other firms' strategy is borne out by the equilibrium solution for the industry. In other words, we assume that firm i would adjust its perceptions (values of k_{G-i}, m_{G-i}) until they explain the actual investment, y_{G-i}, in Y by all firms other than i. That is,

$$k_{G-i} + m_{G-i}y_i^* = y_{G-i}^*. \tag{12}$$

Hence,

$$k_{G-i} = y_G^* - (1 + m_{G-i})y_i^* \tag{13}$$

where y_i^* and y_G^* are optimal investments of firm i and the group G respectively. Using (13) in (11), and simplifying, we can rewrite y_i^* as

$$y_i^* = R_i + \alpha_{X_i} - \frac{1}{(1+m_E)}\cdot\frac{\beta_{X_i}\alpha_{Y_i}}{\beta_{Y_i}(1+m_{G-i})} - \frac{k_E}{(1+m_E)}\cdot\frac{\beta_{X_i}}{\beta_{Y_i}(1+m_{G-i})} - \frac{\beta_{X_i}}{\beta_{Y_i}(1+m_{G-i})}y_G^*. \tag{14}$$

Adding (14) for all i and simplifying we get the industry's individually rational equilibrium allocation to basic research, y_G^* as

[3] Some scholars question the assumption of a fixed amount of investable resources. They point out that, for most firms, investable resources are flexible. If worthy projects are available, additional funds could be raised. Our approach requires that such project-specific borrowing be accounted for in the benefits and costs of the project, but not in the investment part. Secondly, to the extent that administrative practices such as giving each department a fixed budget and letting it decide how to spend it exist, the assumption may be quite realistic.

One could, of course, build a model that accounts for capital rationing through differences in the cost of capital and avoids the use of a budget constraint. However, here we want to retain all of Joglekar and Hamburg's (1983) assumptions except their assumption of independence of benefits.

$$y_G^* = \frac{1}{1+\mu}\left\{R_G + V - \frac{W}{1+m_E} - \frac{\mu k_E}{1+m_E}\right\} \quad \text{where} \tag{15}$$

$$\left.\begin{aligned} \mu &= \sum_{i\in G} \frac{\beta_{X_i}}{\beta_{Y_i}(1+m_{G-i})}, \\ V &= \sum_{i\in G} \alpha_{X_i}, \\ W &= \sum_{i\in G} \frac{\beta_{X_i}\alpha_{Y_i}}{\beta_{Y_i}(1+m_{G-i})}. \end{aligned}\right\} \tag{16}$$

Note that under our assumptions, μ, V and W will each have nonnegative values.

y_T^*, the total equilibrium investment (including government's investment) in Y, is given by

$$y_T^* = \frac{1}{1+\mu}\left[k_E + (1+m_E)(R_G + V) - W\right]. \tag{17}$$

Equations (9) to (17) would be valid only if $y_T^* \geq \pi$. Otherwise, one would obtain the solution in (8).

As Joglekar and Hamburg (1983) noted, in this model also, one of the original parameters, k_{G-i}, is eliminated in the final equilibrium. The reason is that by condition (13) a given value of m_{G-i} uniquely determines the equilibrium k_{G-i} (and vice versa). The important thing to realize is that the condition $k_{G-i} \geq 0$ must still be valid. In fact, when benefits of X and Y are interdependent, each firm has an incentive to allocate some resources to Y so as to ensure adequate payoff from its investment in X. Thus, in our situation, there is a strong presumption that k_{G-i} will be strictly greater than zero. Then by (13) it follows that

$$(1 + m_{G-i}) < y_G^*/y_i^*. \tag{18}$$

Let us now try to compare these conditions of an individually rational equilibrium with the Pareto optimal solution. Olson (1971, p. 27) points out that Pareto optimality will be achieved when members of a group share every marginal dollar of investment in Y in proportion to the marginal increases in their respective RAVs. That is,

$$\frac{\partial \mathrm{RAV}_i}{\partial y_i} = \frac{\partial \mathrm{RAV}_j}{\partial y_j} = \frac{\partial \mathrm{RAV}_G}{\partial y_G} = \frac{\partial \sum_j \mathrm{RAV}_j}{\partial y_G} \quad \text{or} \tag{19}$$

$$\beta_{Y_i}(\alpha_{X_i} + x_i)^{\beta_{X_i}}(\alpha_{Y_i} + y_T)^{\beta_{Y_i}-1}(1+m_E)(1+m_{G-i}) = \sum_{j\in G} \beta_{Y_j}(\alpha_{X_j} + x_j)^{\beta_{X_j}}(\alpha_{Y_j} + y_T)^{\beta_{Y_j}-1}(1+m_E) \quad \text{or,} \tag{20}$$

$$\beta_{Y_i}(\alpha_{X_i} + x_i)^{\beta_{X_i}}(\alpha_{Y_i} + y_T)^{\beta_{Y_i}-1} = \frac{1}{(1+m_{G-i})}\sum_{j\in G} \beta_{Y_j}(\alpha_{X_j} + x_j)^{\beta_{X_j}}(\alpha_{Y_j} + y_T)^{\beta_{Y_j}-1}. \tag{21}$$

Adding both sides of (21) for all $i \in G$, and eliminating common factors, we get

$$\sum_{i\in G} \frac{1}{1 + m_{G-i}^{**}} = 1 \tag{22}$$

where m_{G-i}^{**} is the Pareto optimal matching rate for firm i.

It follows that at the Pareto optimality each firm perceives $k_{G-i}^{**} = 0$. In other words, each firm perceives a pure matching behavior on the part of other firms. If there is no government intervention (i.e., if $k_E = 0$, and $m_E = 0$), the group-rational (Pareto Optimal) investment y_G^{**} for the industry would be

$$y_G^{**} = \frac{1}{1+\mu^{**}}(R_G + V - W^{**}) \qquad \text{where} \tag{23}$$

$$\left.\begin{aligned} \mu^{**} &= \sum_{i \in G} \frac{\beta_X}{\beta_{Y_i}(1 + m_{G-i}^{**})}, \\ V &= \sum_{i \in G} \alpha_{X_i}, \\ W^{**} &= \sum_{i \in G} \frac{\beta_{X_i}\alpha_{Y_i}}{\beta_{Y_i}(1 + m_{G-i}^{**})}. \end{aligned}\right\} \tag{24}$$

Since $k_{G-i}^{**} = 0$, by (1) it follows that

$$1 + m_{G-i}^{**} = y_G^{**}/y_i^{**} \qquad \text{for all } i. \tag{25}$$

From conditions (18), (22), and (25) we conclude that under Pareto optimality the perceived matching rate attains its maximum value for each firm. That is,

$$m_{G-i}^{**} > m_{G-i} \qquad \text{for all } i, \tag{26}$$

where m_{G-i} represents any value for others' matching rate that i can feasibly perceive.

When there is no government intervention (i.e., $k_E = m_E = 0$), y_G^* is given by the modified form of (15),

$$y_G^* = [1/(1+\mu)](R_G + V - W). \tag{27}$$

Now, because of conditions (23) and (27), it is obvious that

$$y_G^* < y_G^{**}. \tag{28}$$

Thus, our model confirms that the individually rational group investment in Y will be smaller than the group rational group investment in Y. In general, individually rational group investment will be substantially smaller than the Pareto optimal group investment in basic research for several reasons detailed in Joglekar and Hamburg (1983, p. 1007). Our analysis supports Olson's (1971) position that voluntary groups will fail to allocate adequate resources to activities whose benefits are inappropriable. It strengthens Arrow's (1962) conclusion that a free enterprise system will invest suboptimally in basic research, by indicating the truth of that proposition even when firms in an industry are allowed to cooperate with one another on a voluntary basis, and when each firm's benefits from the appropriable and inappropriable activities are interdependent.

We take this opportunity to comment on the challenge posed by recent works in supergame (i.e., hypergame and metagame) theory to the logic of Arrow (1962), Olson (1971), and Joglekar and Hamburg (1983), (1986). Basically, the argument of works in supergame theory including Bennett (1980), Evans and Harris (1982), Hofstadter (1983) and Thompson (1984) can be paraphrased as: "Over repeated trials, or in anticipation of repeated trials, of a resource allocation game, the value of the matching rate (m_{G-i}) will go up even in a voluntary group of firms; consequently, underinvestment in inappropriable activities will be wiped out."

The primary basis for this argument lies in some laboratory experiments and computer tournaments using the well known Prisoner's Dilemma from game theory. Axelrod's

(1984) *The Evolution of Cooperation* is prototypical of this argument. As Axelrod describes it,

> In the Prisoner's Dilemma game, there are two players. Each has two choices, namely cooperate or defect. Each must make the choice without knowing what the other will do. No matter what the other does, defection yields a higher payoff than cooperation. The dilemma is that if both defect, both do worse than if both had cooperated. (pp. 7–8).

In other words, in Prisoner's dilemma, defection is the individually rational choice, while cooperation is the Pareto optimal choice. Axelrod (1984) recognizes that if the game is played a known finite number of times, the players have no incentive to cooperate. However, he argues that when an indefinite number of iterations are considered, if individuals have sufficiently large stake in their future interactions, cooperation can emerge. Axelrod (1984) assumes that while a single player may be interacting with many others, he is interacting with them one at a time. The player is also assumed to recognize another player and to remember how the two of them have interacted so far. Under these assumptions, Axelrod finds that even in a world of unconditional defection, cooperation can evolve from small clusters of individuals who base their cooperation on reciprocity, i.e., a TIT for TAT strategy. Such cooperation can thrive in a world where many different strategies are being tried, and it can protect itself from invasion by less cooperative strategies.

Clearly, Axelrod's (1984) logic could be applied to the problem of allocation of resources to basic research only if pairs of firms in an industry were reacting to one another's allocations. This seems unrealistic since the relevant industry consists of not only the established firms but also potential new entrants who may be difficult to identify, let alone reward or punish for their actions. Secondly, whenever three or more firms are involved, even if a given firm reciprocates each of the remaining firm's allocations, without explicit communication, others may fail to notice the reciprocity involved based on their knowledge of the firm's *total* allocation to basic research.

Finally, in a research allocation situation, even when only two firms are involved, a given firm may decide to allocate any amount of resources on a continuum from zero up to its Pareto optimal level. In other words, a firm may display *varying degrees of cooperation*. Prisoner's dilemma does not recognize such a continuum of actions. It requires discrete strategies: cooperation or defection. Clearly, Prisoner's dilemma is an inappropriate model to analyze an industry's allocation of resources to basic research.[4]

In view of the above discussion, our modeling approach seems far more realistic. We simply assume that Firm i perceives that other firms together contribute k_{G-i} to Y regardless of what i does, and in addition, match every dollar of i's contribution to Y with m_{G-i} dollars. In the homogeneous industry case, Joglekar and Hamburg (1986) found, and our analysis here confirms (see §4.5), that as m increases, underinvestment in basic research decreases. However, any conclusion about whether m does or does not increase over repeated trials of a resource allocation game would require solid empirical evidence.

Our models are not iterative. We do not consider repeated trials. We do assume, however, that in any specific resource allocation trial, in the equilibrium, the perceived value of m also is its actual value. This assumption is equivalent to Harsanyi's (1977) principle of mutually expected rationality. While our approach clearly represents a simplification, it is a justifiable simplification.

Finally, we recognize that Axelrod (1984) represents only one argument why supergame theorists believe that, in the long run, voluntary cooperation may be sufficient to overcome the problem of underinvestment in inappropriable goods. However, Evans and Harris

[4] Olson's (1986) review of Axelrod's (1986) book highlights several of these shortcomings of using Prisoner's Dilemma to understand the general problem of provision of public goods such as basic research.

(1982) find it necessary to qualify their conclusion about the long-run optimality of investment in inappropriable goods by stating that

> In certain circumstances (e.g., epidemics and foreign invasions) any learning lag at all may well be unacceptable. Who among us wishes to allow the loss of life and liberty in order to confirm that voluntary cooperation would be sufficient the second time such irreversible events take place? (p. 149).

We believe there are many other circumstances in which society may not want to wait for the second time around.

Thus, even if the supergame theorists are right, and the problem of underinvestment in basic research can be resolved through voluntary action in the long run, in the short run there may be legitimate desires for government intervention to promote investment in basic research. The questions are what *kinds* of industries would need relatively greater governmental support and what *kinds* of support would be effective and efficient.

Joglekar and Hamburg (1983), (1986) addressed precisely those questions. They assumed that a firm's benefits from the relevant (industry-wide) basic research were independent of its benefits from its appropriable investments. Here we verify their conclusions assuming that these benefits are *interdependent*.

3. An Evaluation of Government Policies

As Joglekar and Hamburg (1983) did, we now assess the effectiveness and efficiency of:

(a) the provision of a matching subsidy, m_E, and

(b) the provision of "seed money", k_E.

With government intervention, the total investment in Y is given by (17), and designated by $y^*_{T|E}$ to emphasize the presence of government intervention. Following Joglekar and Hamburg (1983), we assume that government wants $y^*_{T|E}$ to be close to y^{**}_G of (23). One way of accomplishing this is by having government spend the desired amount all by itself. However, government's real objective is to increase the industry's investment as much as possible, without creating an unwarranted burden in terms of tax dollars. Equation (15) gives the industry's investment in Y under government intervention. We represent it by $y^*_{G|E}$ to emphasize the presence of external intervention. $y^*_{G|E} - y^*_G = N_{G|E}$ gives the net effect of government intervention on the industry's investment. We say that an instrument is effective[6] only if $N_{G|E}$ is positive, and the corresponding $y^*_{T|E}$ does not exceed y^{**}_G.

When we have two or more effective instruments, we are further concerned with the efficiency (e) of the tax dollar under each. Government's cost of an instrument is $C_E = y^*_{T|E} - y^*_{G|E}$. The ratio $e = N_{G|E}/C_E$ measures the efficiency of an instrument.

The Effectiveness of Seed Money

If we assume that the benefit function, risk aversion and perceived cooperation in an industry are not affected by the presence of government intervention, we see that

$$\partial y^*_{G|E}/\partial k_E = -\mu/[(1 + m_E)(1 + \mu)] \qquad (29)$$

is negative. Thus, when all firms have noncorner solutions, industry's investment in Y is a decreasing function of government seed money. Since $N_{G|E}$ is also negative, it follows that provision of seed money is not an effective measure of government intervention. In fact, we must say that *the provision of seed money is counterproductive* since it reduces the industry's investment.[5] This is exactly the result obtained by Joglekar and Hamburg

[5] Some economists consider our definition of an *effective policy instrument* to be misleading. They point out that if there is a less than Pareto-optimal amount of the relevant research, a government program that increases

(1983) assuming independence of benefits from appropriable and inappropriable investments. Our analysis confirms the result even when such benefits are interdependent.

As Joglekar and Hamburg (1983) found, there is one case where k_E is effective, namely when Y is lumpy, and in the absence of government intervention, each firm is expected to have a "corner-point-solution" (8). In this case,

$$y_G^* = 0. \tag{30}$$

However, because

$$\partial y_{T|E}^* / \partial k_E = 1/(1+\mu) \tag{31}$$

is positive, with positive value for k_E, government could see to it that

$$y_{T|E}^* \geq \pi \quad \text{and} \tag{32}$$

$$y_{G|E}^* > 0. \tag{33}$$

In fact, in this case, if government wants to use the instrument of seed money alone, ($k_E > 0$, $m_E = 0$) its optimal choice for the value of k_E would be such that

$$y_{T|E}^* = [1/(1+\mu)][k_E + R_G + V - W] = \pi. \tag{34}$$

Beyond this point, a marginal increase in k_E would reduce the industry's investment in Y. Under this optimal choice, the cost to the government would be

$$C_E = k_E = (1+\mu)\pi - R_G - V + W \tag{35}$$

and the net increase in the industry's investment would be

$$N_{G|E} = y_{G|E}^* - y_G^*$$
$$= y_{G|E}^*, \qquad \text{since} \qquad y_G^* = 0. \tag{36}$$

Using (15) and remembering that $m_E = 0$, we have

$$N_{G|E} = [1/(1+\mu)][R_G + V - W - \mu k_E]. \tag{37}$$

Using the value of k_E in (35), then

$$N_{G|E} = R_G + V - W - \mu\pi. \tag{38}$$

Therefore, the efficiency of the proposed seed money is

$$e = \frac{N_{G|E}}{C_E} = \frac{R_G + V - W - \mu\pi}{(1+\mu)\pi - R_G - V + W}. \tag{39}$$

This efficiency can be compared with the efficiency of other effective instruments for the lumpy case. But for the moment we confirm Joglekar and Hamburg's (1983) conclusion that except when Y is lumpy (i.e., $\pi > 0$) provision of seed money is not an effective instrument of government intervention.

The Effectiveness of Matching Subsidies

Assuming that the benefit functions, risk aversion and perceived cooperation in an industry are not affected by the existence of government intervention in the all-noncorner-

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total expenditure can be desirable from the society's point of view, even if private expenditure should fall. While technically these economists are right, our philosophy is that a government's role is to encourage private contribution to public good not substitute for it. This is why we consider a government policy to be "not effective", or *counterproductive*, if it reduces private expenditures on research that is already suboptimally funded. Insofar as our meanings of terms like *effective* and *counterproductive* are clearly defined, no one should be misled.

solutions case, the partial derivative of $y^*_{G|E}$ [the same as y^*_G of (15)] w.r.t. m_E is positive as below.

$$\frac{\partial y^*_{G|E}}{\partial m_E} = \frac{1}{(1+m_E)^2} \cdot \frac{1}{(1+\mu)}(W + \mu k_E). \tag{40}$$

That is, m_E is effective in inducing additional investment from the industry.

Assuming government is considering the use of only the matching subsidy (i.e., $k_E = 0$, $m_E > 0$), the net increase in industry's investment with a given m_E will be

$$\begin{aligned} N_{G|E} &= y^*_{G|E} - y^*_G \\ &= \frac{m_E}{(1+\mu)(1+m_E)} W, \end{aligned} \tag{41}$$

while the cost of the instrument to the government will be

$$\begin{aligned} C_E &= m_E y^*_{G|E} \\ &= \frac{m_E}{(1+\mu)}\left[R_G + V - \frac{1}{(1+m_E)} W\right]. \end{aligned} \tag{42}$$

Hence, the efficiency of a matching subsidy, in the all-noncorner-solution case will be

$$e = \frac{N_{G|E}}{C_E} = \frac{W}{(1+m_E)(R_G+V) - W}. \tag{43}$$

As Joglekar and Hamburg (1983) did, we also carried out numerical calculations with reasonable values for the parameters in our model. We confirm that, even in our model, the cost efficiency of a matching subsidy is often quite low. Particularly when government chooses a matching rate that would completely eliminate suboptimality of investment (i.e., such that $y^*_{T|E} = y^{**}_G$), cost efficiency is a small fraction, and government would spend many times the amount of increase it would induce in the industry's investment in Y. Thus, a matching subsidy is an effective but not very efficient instrument of government intervention for the all-noncorner-solution case. This is also true as long as some firms are operating at noncorner points in the absence of government intervention.

Note that the above analysis assumes that a matching subsidy is available for the *entire investment* of an industry in basic research. As Joglekar and Hamburg (1983, p. 1010) noted, *a marginal matching subsidy*, available only beyond the minimal level (y^*_G), may be not only cost-efficient but also cost-effective.

When the investment opportunity is lumpy, and industry investment in basic research is simply nonexistent without government intervention (i.e., the all-noncornerpoint-solutions case), both the provision of seed money and the provision of a matching subsidy are effective instruments of intervention. Let us now compare the efficiencies of these two measures in the lumpy case. We have already defined the optimal k_E and its efficiency. Now consider that, with $k_E = 0$, government chooses a value of m_E such that

$$y^*_{T|E} = \frac{(1+m_E)(R_G+V) - W}{(1+\mu)} = \pi. \tag{44}$$

Then, the efficiency of the proposed matching subsidy is

$$e = \frac{N_{G|E}}{C_E} = \frac{1}{m_E} = \frac{R_G + V}{(1+\mu)\pi - R_G - V + W}. \tag{45}$$

From (39) and (45) it is easy to see that in the lumpy case, *provision of a matching subsidy is a more efficient instrument than the provision of seed money.*

As Joglekar and Hamburg (1983) observed, given that $\partial y^*_{G|E}/\partial k_E$ is negative and $\partial y^*_{G|E}/\partial m_E$ is positive, one can construct an optimal instrument of government intervention involving a flat tax and a matching subsidy such that both the industry's allocation to Y and the total investment in Y would be positively affected. However, we do not present the formulas for such an optimal instrument here because Joglekar and Hamburg (1983) noted important practical problems in its potential implementation.

4. Analysis of the Case of Homogeneous Industry

Joglekar and Hamburg (1986) focused on the homogeneous industry case of their 1983 model to investigate the effects of several relevant factors upon an industry's allocation of resources to basic research, and derived several important policy conclusions. We now verify whether those conclusions would be valid even when a firm's benefits from appropriable and inappropriable investments are interdependent.

We consider a homogeneous industry that is free of any government intervention. That is,

$$k_E = 0, \qquad m_E = 0. \tag{46}$$

By definition, in a homogeneous industry all firms are identical. Hence,

$$\left.\begin{aligned} \alpha_{X_i} &= \alpha_{X_j} = \alpha_X, \\ \beta_{X_i} &= \beta_{X_j} = \beta_X, \\ \alpha_{Y_i} &= \alpha_{Y_j} = \alpha_Y, \\ \beta_{Y_i} &= \beta_{Y_j} = \beta_Y, \\ m_{G-i} &= m_{G-j} = m, \\ R_i &= R_j = R = R_G/g. \end{aligned}\right\} \tag{47}$$

In this case, for notational ease, let us further define

$$\beta_X/\beta_Y = \theta. \tag{48}$$

Remember that since we expect $\beta_X \geq \beta_Y$, we expect $\theta \geq 1$.

Then, substituting (46), (47) and (48) in (15) and (16), we obtain

$$y^*_G = \frac{(1+m)}{(1+m+g\theta)}\left\{R_G + g\alpha_X - \frac{g\theta\alpha_Y}{1+m}\right\}. \tag{49}$$

We also note that in the homogeneous industry case, condition (22) implies that

$$1 + m^{**} = g. \tag{50}$$

Consequently, the Pareto optimal industry investment in basic research is given by

$$y^{**}_G = \frac{1}{(1+\theta)}\{R_G + g\alpha_X - \theta\alpha_Y\}. \tag{51}$$

Thus, $S = [(y^{**}_G - y^*_G)/y^{**}_G]$, the degree of suboptimality of the industry's actual investment in basic research can be shown to be given by

$$S = \frac{\theta(g-m-1)}{(g\theta+m+1)}\cdot\frac{(R_G + g\alpha_X + \alpha_Y)}{(R_G + g\alpha_X - \theta\alpha_Y)}. \tag{52}$$

Let us now analyze the sensitivity of this degree of suboptimality to each of the variables involved in our model so as to identify situations in which the need for government intervention is relatively greater. As Joglekar and Hamburg (1986) did, we assume that government wants to minimize this degree of suboptimality.

4.1. *Sensitivity to the General Competitive Advantage of the Firm*

As explained earlier, in this model α_X reflects (although, it does not exactly measure) a firm's general competitive advantage. One can easily see that

$$\frac{\partial S}{\partial \alpha_X} = \frac{\theta(g - m - 1)}{(g\theta + m + 1)} \cdot \frac{[-g(1 + \theta)\alpha_Y]}{(R_G + g\alpha_X - \theta\alpha_Y)^2}. \tag{53}$$

Under our assumptions, this is negative. Thus, other things equal, the greater the general competitive advantage an average firm in the industry has, the smaller would be the suboptimality of group investment in Y, and hence the smaller would be the need for government intervention.

Note that α_{X_i} reflects the benefit that firm i obtains even when it decides to invest no new capital in its appropriable investment opportunity. The value of α_{X_i} depends upon such things as a firm's patent position, current capacity, market share, market niche, etc. In a homogeneous industry, by definition, all α_{X_i} are equal ($=\alpha_X$). Still, in homogeneous industries where interfirm competition is relatively less, e.g., due to regionally segmented monopolies in the market, α_{X_i} is likely to be higher than in homogeneous industries with intense interfirm competition. Our result confirms Joglekar and Hamburg's (1986) conclusion that *an industry that manifests relatively more intense interfirm competition will operate at a higher degree of suboptimality of investment in basic research, and would have a higher need for government intervention.*

4.2. *Sensitivity to the Firm's Advantage from Industry's Past Investment in Basic Research*

α_Y reflects the a priori advantage each firm in an industry enjoys because of past basic research of the industry as a whole. Differentiating S w.r.t. α_Y we obtain

$$\frac{\partial S}{\partial \alpha_Y} = \frac{\theta(g - m - 1)}{(g\theta + m + 1)} \frac{[(1 + \theta)(R_G + g\alpha_X)]}{(R_G + g\alpha_X - \theta\alpha_Y)^2}. \tag{54}$$

Under our assumptions $\partial S/\partial \alpha_Y$ will be positive. Thus, other things being equal, *the greater the general competitive advantage of an industry over other industries, the greater will be the degree of suboptimality of investment in Y.*

Note that α_Y reflects the benefit each firm in the industry obtains even when the industry as a whole invests no new capital in its basic research. In addition to the industry's past investment in its basic research, a number of other factors can influence the value of α_Y. For a rich discussion of those factors and the pertinent policy conclusions, see Joglekar and Hamburg (1986, pp. 228–229).

4.3. *Sensitivity to the "Marginal Utilities" of a Dollar Invested in Appropriable Versus Inappropriable Opportunities*

While introducing the variables in this model, we said that β_X reflects (but does not exactly measure) the marginal utility of a dollar invested in X. However, because of our assumption of interdependence of benefits, the actual effects of changes in β_X on a firm's allocation to the types of investments, are likely to be much more complex. Joglekar and Hamburg (1986) with their assumption of independence of benefits found that, other things equal, the larger the expected benefits per dollar of investment in X, the greater was the degree of suboptimality of investment in Y. Intuitively, we do not expect such a clear cut finding for the β_X under our assumption of interdependence of benefits. Joglekar and Hamburg (1986) also found that the larger the expected benefits per dollar of investment in Y, the smaller was the degree of suboptimality of investment in Y. Again we do not anticipate such a clear cut sensitivity to our β_Y, the variable that reflects the "marginal utility" of a dollar invested in Y. We have defined θ as the ratio of β_X/β_Y. Under our assumptions,

$$\frac{\partial S}{\partial \theta} = \frac{(g-m-1)}{(g\theta+m+1)^2} \frac{(R_G+g\alpha_X+\alpha_Y)[(m+1)(R_G+g\alpha_X)+g\theta^2\alpha_Y]}{(R_G+g\alpha_X-\theta\alpha_Y)^2} \tag{55}$$

is clearly positive. In other words, despite our doubts, other things remaining the same, as β_X increases, or β_Y decreases, the degree of suboptimality of investment in Y will increase. Thus, even in the case of interdependent benefits, to the extent that β_X and β_Y reflect the marginal utilities of investments in X and Y respectively, Joglekar and Hamburg's (1986) findings are confirmed.

4.4. *Sensitivity to Investable Resources of Each Firm*

Remembering that $R_G = gR$, we obtain

$$\frac{\partial S}{\partial R} = \frac{(g-m-1)}{(g\theta+m+1)} \cdot \frac{(-g\alpha_Y)(1+\theta)}{(R_G+g\alpha_X-\theta\alpha_Y)^2}. \tag{56}$$

Clearly, $\partial S/\partial R$ is negative under our assumptions. Thus, we confirm Joglekar and Hamburg's (1986) finding that, other things equal, the larger the investable resources of the average firm in an industry, the smaller is the degree of suboptimality of investment in basic research. Hence, government should focus on promoting basic research in industries that are strapped for cash and capital rather than those that have relatively plentiful capital.

4.5. *Sensitivity to the Perceived Matching Rate for a Firm's Investment in Basic Research*

We have defined m as the rate at which each firm expects others together to match its investment in basic research. We have also said that for any meaningful results, m is between -1 and $g-1$. Joglekar and Hamburg (1983), (1986) implied that m is larger when an industry displays a cooperative climate. However, Joglekar and Hamburg (1989) suggest that m may also be larger when there is strong inter-firm rivalry in an industry. We find merit in both of these interpretations. As such, although Joglekar and Hamburg (1986) talked about the sensitivity of S to the cooperative/exploitative climate in an industry, here we simply talk about the sensitivity of S to m, the perceived matching rate. In our model,

$$\frac{\partial S}{\partial m} = \frac{-g(\theta+1)\theta}{(g\theta+m+1)^2} \cdot \frac{(R_G+g\alpha_X+\alpha_Y)}{(R_G+g\alpha_X-\theta\alpha_Y)} \tag{57}$$

is negative as in Joglekar and Hamburg (1986). Thus, we confirm that as the perceived matching rate increases, other things being equal, the degree of suboptimality of investment in basic research decreases. However, for a fuller interpretation of this result, the reader should refer to all of Joglekar and Hamburg's prior works on this topic (1983), (1986), (1987) and (1989).

Based on the numerical examples we constructed to verify the integrity and the meaningfulness of this model, we can further confirm that particularly when the Cournot assumption is valid (i.e., $m = 0$), S, the magnitude of suboptimality, is substantial in small groups, and larger yet in large groups. This conclusion is all the more important insofar as Olson (1971) suggests that the larger the group the smaller is the m. Let us discuss the sensitivity of S to the number of firms, g, more formally in the following section.

4.6. *Sensitivity to the Group Size*

As Joglekar and Hamburg (1986) point out, the study of sensitivity to group size is more complex than the study of sensitivity to the other variables in the model. This is because several other variables may be dependent on group size. We already alluded to the possibility that in larger groups m is likely to be smaller. If we consider a situation where the group size in an industry is increasing because of new entrants in a static market, one could argue that since the market size is fixed, the industry's total investable resources, R_G, would be also independent of g. We shall comment on such situations later, but let us first consider the case when θ, m, R, α_X and α_Y are all independent of g.

In this case, from (52), S can be written as

$$S = \frac{\theta(g - m - 1)}{(g\theta + m + 1)} \cdot \frac{[g(R + \alpha_X) + \alpha_Y]}{[g(R + \alpha_X) - \theta\alpha_Y]} . \tag{58}$$

Differentiating S w.r.t. g, we obtain,

$$\frac{\partial S}{\partial g} = \frac{\theta(1 + \theta)[g^2(R + \alpha_X) + (1 + m)\alpha_Y][(1 + m)(R + \alpha_X) - \theta\alpha_Y]}{(g\theta + m + 1)^2[g(R + \alpha_X) - \theta\alpha_Y]^2} . \tag{59}$$

From (49) we know that in a noncorner solution case (i.e., when $y_G^* > 0$),

$$R + \alpha_X > \frac{\theta\alpha_Y}{(1 + m)} \qquad \text{i.e.,} \tag{60}$$

$$(1 + m)(R + \alpha_X) > \theta\alpha_Y. \tag{61}$$

From (61) it is clear that the numerator of (59) will be greater than 0. Consequently, $\partial S/\partial g > 0$. Thus, in this case, as the group size increases, the degree of suboptimality of investment in basic research also increases.

Remember that when R and m are not functions of g we have shown that $\partial S/\partial R < 0$ and $\partial S/dm < 0$. Thus, when either R or m is a decreasing function of g, clearly $\partial S/\partial g$ will be positive. Consequently, we can confirm Joglekar and Hamburg's (1986) finding that, *in general*, an industry consisting of a larger number of firms will fall farther short of its Pareto optimal investment in basic research than an industry consisting of a small number of firms.

Clearly, our analysis contradicts the suggestions by Chamberlin (1974), Hardin (1971) and Frohlich and Oppenheimer (1970) that voluntarily cooperating industry groups may invest socially optimal amounts in their inappropriable activities (e.g. basic research) regardless of the group size involved. Instead, our analysis confirms Olson's (1971) position that such voluntary groups will fail to allocate adequate resources to inappropriable activities, particularly so in larger groups. It follows that *there is a greater need for government intervention when the total number of firms in an industry is large.*

4.7. *Sensitivity to Risk Aversion on the Part of Each Firm*

Using our model of interdependent benefits, we have confirmed practically all of the policy conclusions at which Joglekar and Hamburg (1983), (1986) arrive. One important exception is Joglekar and Hamburg's (1986) conclusion that a risk averse industry calls for less government intervention than an industry with relatively lesser risk aversion. While this is an important conclusion to verify insofar as it contradicts a popular belief, my attempts at models that assume that a firm's joint (interdependent) benefits from its appropriable investment and the industry-wide inappropriable investment are randomly distributed with an assumed form of distribution and that the firm is faced with a separate form of utility function led to considerable algebraic complexity. To keep things simple, in this model I compromised by simply defining a firm's risk adjusted value from the investments in the two types of opportunities. As a result of that compromise, I cannot verify Joglekar and Hamburg's (1986) conclusion about the sensitivity to risk aversion. An important goal for my future research is to verify that conclusion. Other researchers are encouraged to do the same.

5. Discussion and Conclusion

Policy makers are often reluctant to follow up on the findings and conclusions of a model based (essentially theoretical) study. This is quite understandable since these findings and conclusions are simply as valid as the assumptions underlying the model. Since there were indeed a series of assumptions underlying Joglekar and Hamburg's earlier (1983), (1986) works, it is no surprise that policy makers have not implemented their findings. However, by now, it seems clear that their conclusions cannot be simply disregarded because they are theoretical. I have shown that a number of their findings, albeit model based, are valid under alternative sets of assumptions. Their earlier (1983) model assumed *independence* of a firm's benefits from its appropriable investment and from the industry-wide inappropriable investment, each distributed *exponentially*. In Joglekar and Hamburg (1987), they showed that even when the benefits are (independent, but) *normally* distributed, most of the earlier findings hold. Here, we have seen that when the benefits are *interdependent*, most of the earlier findings hold.

It should be admitted here that the types of models we have constructed cannot deal with some larger issues. For example, our models cannot answer whether the nonoptimality introduced by extra taxation to subsidize basic research is smaller or larger than the nonoptimality of too little basic research.[6] Nor do they address administrative problems and related issues of how a government program works out in practice. However, as policymakers grapple with those larger issues, they may be comforted in knowing that at least, the following conclusions are robust:

Unaided industry allocation to basic (inappropriable) research is suboptimal even when firms are permitted to conduct basic research jointly (but voluntarily). Industries with one or more of the following characteristics are likely to have a higher degree of suboptimality of investment in basic research:

(a) *Industries involving a large number of firms,*

(b) *Industries with intense interfirm competition, and, intense rivalry*

(c) *Industries with a lesser threat from other industries, and*

(d) *Industries with smaller amounts of investable resources.*

Clearly, industries displaying these characteristics deserve more government assistance in their basic research programs. At the same time, our analysis has shown that provision of seed money is generally counterproductive, while the provision of a matching subsidy is not cost-efficient in increasing industry's allocation to basic research.[7]

[6] In constructing their "optimal instrument of government intervention," Joglekar and Hamburg (1983) sought to tax the very companies that were suboptimally allocating their resources to basic research and planned to return those taxes to the same companies through matching subsidies. If such an optimal instrument were adopted, this type of question would not be relevant.

[7] I wish to thank Professor Morris Hamburg of the Wharton School for his meticulous examination of my mathematics, and for his many suggestions for improvements in the earlier drafts. Of course, the responsibility for any remaining errors is entirely mine.

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