Concepts, Theory, and Techniques

INDUSTRY RESOURCE ALLOCATION TO BASIC RESEARCH UNDER NORMALLY DISTRIBUTED BENEFITS*

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ABSTRACT

The failure of a free market system to attain socially optimal allocation of resources to research and development (R&D) is a generally recognized problem. However, we are just beginning to understand the types of R&D activities that receive relatively serious underinvestment from specific types of industries and the types of governmental intervention strategies that are likely to be effective and efficient in the correction of that underinvestment. Recently, Joglekar and Hamburg [16] [17] sought answers to these types of questions using models of the resource allocation behavior of firms considering investment in basic research related to their industry. It was assumed that the firms' benefits were exponentially distributed. In the present article, such benefits are assumed to be normally distributed, and an attempt is made to verify the earlier conclusions and policy implications of [16] and [17]. The results are similar for these two substantially different types of distribution, but the earlier conclusions and policy implications are clarified, qualified, and extended.

Subject Areas: Decision Analysis, Government, R&D, Resource Allocation.

INTRODUCTION

Economists long have contended that a free market system will fail to attain socially optimal allocation of resources to research and development (R&D). Arrow [1] identified two reasons for such underinvestment: (1) the risk inherent in any R&D activity and (2) the inability to appropriate the benefits of many R&D activities.[1]

Not all R&D activities possess the above two characteristics to the same degree. For example, basic research has a high degree of inability to appropriate benefits in addition to some risk; applied R&D activities, on the other hand, are likely to be largely appropriable (under U.S. patent laws) but highly risky.[2] In any case, the presence of either one of these two characteristics is sufficient for underinvestment in a particular R&D activity to take place in a free market system.

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[1]For a detailed discussion of Arrow's argument and some counterarguments, see the text and notes in [16, pp. 997-998].

[2]Strictly speaking, it is not correct to label all basic research as inappropriable or all applied research as appropriable. For a further discussion of this point and an understanding of the use of these terms, see [16, footnote 1].

Arrow's [1] discussion failed to distinguish between underinvestment because of inappropriability and underinvestment because of risk. Arrow also prescribed no governmental policies to correct underinvestment in different types of R&D activities. Our research suggests that the effects of risk and inappropriability on the degree of underinvestment are not necessarily additive—particularly when interfirm cooperation is permitted. It is important to distinguish between *basic* (inappropriable) research and *applied* (appropriable) R&D when analyzing the effectiveness and efficiency of federal policy instruments aimed at stimulating industry investment in R&D. The appropriateness of certain federal policies for stimulating *applied* R&D has been discussed elsewhere [15]. Here we focus on government policies for stimulating *basic* research.

Recently, Joglekar and Hamburg [16] published a model of the resource allocation behavior of a group of firms considering investment in basic research. The analysis showed that unaided industry allocation to basic (inappropriable) research is suboptimal, provision of government seed money generally is counterproductive, and the provision of matching subsidies is not cost efficient in stimulating industry allocation to basic research [16].

In a follow-up article, Joglekar and Hamburg [17] showed that the degree of suboptimality of investment in basic research depends on several industry characteristics. Industries with one or more of the following characteristics are likely to have a higher degree of suboptimality of investment in basic research:

- Industries that involve a large number of firms,
- Industries with intense interfirm competition,
- Industries with less threat from other industries,
- Industries with smaller amounts of investable resources, and
- Industries with smaller degrees of risk aversion.

It was argued that industries displaying these characteristics deserve more government assistance in their basic research programs [17]. Both [16] and [17] assumed that a firm's benefits from its investments (whether or not those benefits were derived from basic (i.e., inappropriable) research or some other appropriable investment opportunity) were exponentially distributed. In justifying the choice of the exponential distribution, Joglekar and Hamburg stated that

> R&D managers would probably approve of the following properties of the exponential distribution in representing anticipated benefits of R&D investments: (a) the exponential distribution rules out the possibility of "negative benefits," (b) it assumes that the likelihood of a particular value of benefits increases as the value increases, with the most likely value (i.e., the mode of the distribution) being zero. However, an exponential distribution also implies that the standard deviation of the distribution is the same as its mean. Most R&D managers may question that implication, often suggesting that the "technical risk" is far smaller. However, considering the commercial risk as well, it seems that benefits of a number of real life R&D investments may be reasonably well represented by an exponential distribution (particularly in the context of our model, since we have imposed only the most reasonable restrictions on the values of the parameters of the distribution). [16, p. 1000]

Clearly the choice of an exponential distribution was arbitrary. In fact the exponential distribution was selected primarily because of its algebraic convenience.

In this paper, therefore, an attempt is made to verify the validity of the conclusions in [16] and [17] in the case where benefits are distributed normally rather than exponentially. The properties of a normal distribution differ substantially from those of an exponential distribution. A normal distribution does not rule out negative benefits. It assumes that benefits are distributed symmetrically around the mean (and the mode) of the distribution, and it permits different values for the mean and standard deviation. Given these differences, it is very encouraging that most of the conclusions using a normal distribution are parallel, if not identical, to those using an exponential distribution. However, it may be noted that the models and analysis presented here help to clarify, qualify, and extend the earlier conclusions.

THE MODEL

A Firm, Its Industry Group, and Its Investment Opportunities

Consider a firm i belonging to a group $G = \{1, 2, \ldots, i, \ldots, g\}$ of g firms. Let i have two investment opportunities X_i and Y. Assume the benefits B_{Xi} of an investment in X_i accrue only to firm i (i.e., X_i represents a purely appropriable investment opportunity for firm i). Therefore, only i is expected to invest an amount x_i in X_i. Let B_{Xi} be distributed normally as

$$f(B_{Xi}) = (2\pi)^{-1/2}\sigma_{Xi}^{-1} \exp - \frac{(B_{Xi} - \mu_{Xi})^2}{2\sigma_{Xi}^2} \tag{1}$$

where μ_{Xi} and σ_{Xi} are respectively the mean and standard deviation of the distribution. Assume that both μ_{Xi} and σ_{Xi} are linearly increasing functions[3] of x_i as below:

$$\mu_{Xi} = \alpha_{Xi} + \beta_{Xi}x_i \tag{2}$$

and

$$\sigma_{Xi} = \gamma_{Xi} + \eta_{Xi}x_i \tag{3}$$

where α_{Xi} represents the mean of the general competitive advantage (or disadvantage) of firm i and γ_{Xi} represents the standard deviation (or risk) of that general

[3]Kamien and Schwartz [18] provide some support for assuming linearly related costs and benefits and also raise some doubts about it (see [16, footnote 6]). Since the available literature does not provide any alternative relationship between research inputs and outputs, we have assumed a linear relationship between R&D expenditures and mean benefits only. We know of no empirical evidence concerning the standard deviation of those benefits. Again, linearity is assumed because of its mathematical simplicity.

competitive advantage. β_{Xi} and η_{Xi} are the amounts of increase in the mean and standard deviation of the benefits-per-dollar increase in the investment x_i, respectively. Assume both of these values are positive, that is, $\beta_{Xi}>0$ and $\eta_{Xi}>0$. The values that α_{Xi} assumes depend on the circumstances of the firm and are unconstrained as to their sign. The measure of standard deviation at $x_i=0$ (γ_{Xi}) is nonnegative.

Let Y represent those basic (i.e., inappropriable) research activities whose benefits fall on the entire group G regardless of who invests in Y. Let y_i be firm i's investment in Y. Let y_{G-i} be the investment in Y by all firms other than i. That is,

$$y_{G-i}=\sum_{j\neq i} y_j \qquad j\in G. \tag{4}$$

Hence y_G, the group's total investment in Y, will be

$$y_G=y_i+y_{G-i}. \tag{5}$$

Further, let y_E be the investment in Y from sources external to the industry (e.g., government). Then the total investment (y_T) in Y from all sources will be

$$y_T=y_E+y_G=y_E+y_i+y_{G-i}. \tag{6}$$

Because of the inability to appropriate the benefits of Y, firm i's benefits (B_{Yi}) depend on this total investment (y_T) in Y. Let B_{Yi} be normally distributed as

$$f(B_{Yi})=(2\pi)^{-1/2}\sigma_{Yi}^{-1}\exp-\frac{(B_{Yi}-\mu_{Yi})^2}{2\sigma_{Yi}^2} \tag{7}$$

where μ_{Yi} and σ_{Yi} are respectively the mean and standard deviation of the distribution. Further assume that basic research is a "lumpy" good; that is, in order for an investment in basic research to yield any meaningful results and consequent economic benfits, y_T must be at least equal to a minimum threshold level Π. Beyond this threshold level, mean benefits and the associated standard deviation both are assumed to be linearly increasing functions of y_T. These parameters are represented as follows:

$$\mu_{Yi}=\begin{cases}\alpha_{Yi} & \text{if } y_T<\Pi\\ \alpha_{Yi}+\beta_{Yi}y_T & \text{if } y_T\geq\Pi\end{cases} \tag{8}$$

and

$$\sigma_{Yi}=\begin{cases}\gamma_{Yi} & \text{if } y_T<\Pi\\ \gamma_{Yi}+\eta_{Yi}y_T & \text{if } y_T\geq\Pi\end{cases} \tag{9}$$

where α_{Yi} represents the mean of the a priori advantage of firm i because of past research in the industry, γ_{Yi} represents the standard deviation of firm i's a priori advantage, and β_{Yi} and η_{Yi} are the amounts of increase in the mean and standard deviation of firm i's benefits-per-dollar increase in the investment y_T, respectively.

Assume $\beta_{Yi}>0$ and $\eta_{Yi}>0$ and that γ_{Yi} is nonnegative. The sign of α_{Yi} is unconstrained since an industry that has neglected its basic research for years in fact may suffer substantial losses in the absence of new basic research. It is implied that the threshold level of investment (Π) is greater than or equal to 0.

As in [16] and [17], firm i's utility function is assumed to be a linear transform of the exponential function.[4] That is,

$$U_i(w_i+B_i)=(1/r_i)(1-\exp(-r_i(w_i+B_i))) \tag{10}$$

where w_i is the initial wealth of firm i and r_i is the risk aversion constant. Assume $r_i>0$.

Risk-Adjusted Benefits

Under the above assumptions, firm i's risk-adjusted value (RAV) of the benefits (B_{Xi}) is given by

$$RAV(B_{Xi})=\mu_{Xi}-\tfrac{1}{2}r_i\sigma^2_{Xi}. \tag{11}$$

That is,

$$RAV(B_{Xi})=\alpha_{Xi}+\beta_{Xi}x_i-\tfrac{1}{2}r_i(\gamma_{Xi}+\eta_{Xi}x_i)^2. \tag{12}$$

It is expected that

$$\frac{\partial RAV(B_{Xi})}{\partial x_i}>0 \tag{13}$$

for the entire feasible range of x_i,

$$0\leq x_i\leq R_i, \tag{14}$$

where R_i denotes the total investable resources at the disposal of firm i.

Conditions (13) and (14) imply that

$$\beta_{Xi}\geq r_i\eta_{Xi}(\gamma_{Xi}+\eta_{Xi}R_i). \tag{15}$$

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[4]For a rationale underlying the exponential utility assumption, a discussion of the meaning of r_i (the risk-aversion constant), and a clear definition of the concept of risk-adjusted value, see [16, pp. 1001-1002] and [7, pp. 249-252].

Similarly,

$$RAV(B_{Yi}) = \mu_{Yi} - \tfrac{1}{2} r_i \sigma^2_{Yi}. \tag{16}$$

That is,

$$RAV(B_{Yi}) = \alpha_{Yi} + \beta_{Yi} y_T - \tfrac{1}{2} r_i (\gamma_{Yi} + \eta_{Yi} y_T)^2 \tag{17}$$

provided $y_T \geq \Pi$, and it is expected that

$$\frac{\partial RAV(B_{Yi})}{\partial y_T} > 0 \tag{18}$$

for the entire feasible range of y_T,

$$0 \leq y_T \leq R_G + y_E, \tag{19}$$

where R_G represents the investable resources available to the group of firms G. Together these conditions imply

$$\beta_{Yi} \geq r_i \eta_{Yi} [\gamma_{Yi} + \eta_{Yi} (R_G + y_E)]. \tag{20}$$

It can be verified that the law of diminishing return holds, as it does in the case of the exponential distribution. That is,

$$\frac{\partial^2 RAV(B_{Xi})}{\partial x_i^2} < 0 \tag{21}$$

and

$$\frac{\partial^2 RAV(B_{Yi})}{\partial y_T^2} < 0. \tag{22}$$

Thus the new assumptions are reasonably realistic.

A Rational Firm's Behavior

Assume a firm's benefits from investments in X_i and Y are independent of each other.[5] A rational firm would want to maximize

[5]The assumption of independent benefits from the two types of investment is justified for our models since the firm's specific investment (X) may or may not be in R&D (see [15, p. 1016]). It is unlikely that a firm's plant investment benefits this year would depend on its investment benefits in basic research (Y) this year. Basic research often is conducted with an eye to improving benefits from

$$F_i = RAV(B_{Xi}) + RAV(B_{Yi}) \tag{23}$$

subject to its budget constraint

$$x_i + y_i = R_i. \tag{24}$$

Since F_i depends on the total investment (y_T) in Y rather than the investment (y_i) by firm i alone, it is imperative that a rational firm make certain assumptions about the resource allocation behavior of other firms. Following Cournot's[6] work on oligopolies, economists often assume that firm i believes the other firms' behaviors are not influenced by its own (i.e., y_{G-i} is independent of y_i). However, another school of thought assumes each firm believes the other firms will match its own investment in Y, positively or negatively. That is, they assume y_{G-i} is an increasing (if the match is positive) or decreasing (if the match is negative) function of y_i. In order to build a general model that can account for either of these two assumptions, we will say that

$$y_{G-i} = \mathrm{k}_{G-i} + \mathrm{m}_{G-i} y_i \tag{25}$$

where k_{G-i} is a nonnegative constant representing the minimal investment in Y by all firms other than i no matter what i does (i.e., $\mathrm{k}_{G-i} \geq 0$) and m_{G-i} is a constant representing their matching rate per dollar of i's investment in Y. The constant m_{G-i} can take on a positive, zero, or negative value depending on the size, structure, and climate (cooperative or exploitative) in an industry. However, m_{G-i} always must be greater than -1, otherwise a firm would have no incentive at all to invest in Y. In industries consisting of large numbers of small firms (where the Cournot assumption may be particularly valid) the value of m_{G-i} is expected to be 0. Even in industries consisting of small numbers of firms, m is likely to be close to 0 because of free-rider behavior. In any case, we expect $\mathrm{m}_{G-i} \leq g-1$. When m_{G-i} equals $g-1$, a firm is assuming that every other firm in the industry will match its investment in basic (inappropriable) research, dollar for dollar. This is a very unrealistic assumption. As shown in equations (44) through (46), the assumptions $\mathrm{k}_{G-i} \geq 0$ and $\mathrm{m}_{G-i} \leq g-1$ are mutually consistent for an industry equilibrium.

future plant investments. Such improvements may be captured through proper valuation of β_{Yi} now and of α_{Xi} and β_{Xi} later. Also, in evaluating a firm's basic research benefits, the industry's total investment is what matters. The firm's own allocation is less important. Consequently, the interdependence of benefits, if any, is likely to be weak.

On the other hand, a basic research activity with *no* appropriable benefits is highly unlikely. A firm that invests in basic research undoubtedly benefits from its scientists' activities, even if their research is largely in the public domain. Until empirical studies confirm that benefits between the two investments are independent, our assumption must be one of convenience. Joglekar [14] explored a model that assumed interdependence of benefits. The results obtained were strikingly different but not contradictory to the results in [16] [17] or those presented here.

[6]For a discussion of Cournot's work and alternative schools of thought about the perceived matching rate, see [20, pp. 25-28].

Furthermore, if the government believes that the industry's investment in basic research will be suboptimal, it may provide encouragement in the form of seed money or matching subsidies or both. The government's investment (y_E) in Y can be represented as

$$y_E = k_E + m_E y_G \tag{26}$$

where k_E is the government's seed money regardless of the industry's investment in basic research and m_E is the government's matching subsidy per dollar of industry investment. We expect $k_E \geq 0$ and $m_E \geq 0$. Given the foregoing assumptions, the total investment (y_T) in Y can be written as

$$y_T = k_E + (1+m_E)\mathrm{k}_{G-i} + (1+m_E)(1+\mathrm{m}_{G-i})y_i. \tag{27}$$

As in [16], if the firm expects that

$$y_T < \Pi \tag{28}$$

its rational choice will be to invest all its resources in X_i. But if we assume

$$y_T \geq \Gamma, \tag{29}$$

a noncorner solution (x_i^*, y_i^*) for firm i would satisfy

$$\frac{\partial F_i}{\partial x_i} = \frac{\partial F_i}{\partial y_i} \tag{30}$$

and

$$x_i^* + y_i^* = R_i. \tag{31}$$

Since

$$\frac{\partial F_i}{\partial x_i} = \beta_{Xi} - r_i \gamma_{Xi} \eta_{Xi} - r_i \eta_{Xi}^2 x_i \tag{32}$$

and

$$\frac{\partial F_i}{\partial y_i} = (\beta_{Yi} - r_i \gamma_{Yi} \eta_{Yi} - r_i \eta_{Yi}^2 y_T) \frac{\partial y_T}{\partial y_i} \tag{33}$$

where

$$y_T = k_E + (1+m_E)[\mathrm{k}_{G-i} + (1+\mathrm{m}_{G-i})y_i] \tag{34}$$

and

$$\frac{\partial y_T}{\partial y_i} = (1+m_E)(1+\mathrm{m}_{G-i}), \tag{35}$$

it can be shown that

$$x_i^* = \frac{r_i\eta_{Yi}^2(1+m_E)^2(1+\mathrm{m}_{G-i})^2R_i - V_{Xi} + V_{Yi}}{r_i[\eta_{Xi}^2 + \eta_{Yi}^2(1+m_E)^2(1+\mathrm{m}_{G-i})^2]} \tag{36}$$

and

$$y_i^* = \frac{r_i\eta_{Xi}^2R_i - V_{Xi} + V_{Yi}}{r_i[\eta_{Xi}^2 + \eta_{Yi}^2(1+m_E)^2(1+\mathrm{m}_{G-i})^2]} \tag{37}$$

where

$$V_{Xi} = \beta_{Xi} - r_i\gamma_{Xi}\eta_{Xi} \tag{38}$$

and

$$V_{Yi} = (1+m_E)(1+\mathrm{m}_{G-i})\{\beta_{Yi} - r_i\gamma_{Yi}\eta_{Yi} - r_i\eta_{Yi}^2[k_E + (1+m_E)\mathrm{k}_{G-i}]\}. \tag{39}$$

Note that

$$V_{Xi} = \left.\frac{\partial F_i}{\partial x_i}\right|_{x_i=0} \tag{40}$$

and

$$V_{Yi} - r_i\eta_{Yi}^2(1+m_E)^2(1+\mathrm{m}_{G-i})^2R_i = \left.\frac{\partial F_i}{\partial y_i}\right|_{y_i=0}. \tag{41}$$

Hence, the condition for a positive value of x_i^* is that the marginal increase in firm i's risk-adjusted value-per-dollar investment increase in X_i should exceed the marginal increase in its risk-adjusted value-per-dollar investment increase in Y when the original investment in X_i is zero.

A similar condition for a positive value of y_i^* also can be identified. As in the exponential case, the point is that meaningful interpretations are available in the normal case for conditions with a noncorner solution.

Industry Equilibrium Assuming Homogeneity

Our discussion of the case of the normal distribution so far has been parallel to discussions that appeared in papers that used the exponential distribution. However, the algebra involved in the normal distribution case is substantially more complex than that in the exponential case. We simplify the algebra in the following discussion by considering only a homogeneous industry.

It may be noted that the analysis of a heterogeneous industry in [14] led to one conclusion—that heterogeneity does not necessarily reduce the degree of suboptimality of investment in Y. For that conclusion to be valid, it is sufficient to demonstrate the existence of one case where heterogeneity increases the degree of suboptimality. This already has been accomplished using the exponential distribution. Thus the fact that our discussion here is restricted to the case of a homogeneous industry does not detract from our objective of testing whether these conclusions depend on the assumed distribution.

The assumption of a homogeneous industry means that

$$\left.\begin{array}{ll} \alpha_{Xi}=\alpha_{Xj}=\alpha_X & \alpha_{Yi}=\alpha_{Yj}=\alpha_Y \\ \beta_{Xi}=\beta_{Xj}=\beta_X & \beta_{Yi}=\beta_{Yj}=\beta_Y \\ \gamma_{Xi}=\gamma_{Xj}=\gamma_X & \gamma_{Yi}=\gamma_{Yj}=\gamma_Y \\ \eta_{Xi}=\eta_{Xj}=\eta_X & \eta_{Yi}=\eta_{Yj}=\eta_Y \\ r_i=r_j=r & R_i=R_j=R \\ \mathrm{k}_{G-i}=\mathrm{k}_{G-j}=\mathrm{k} & \mathrm{m}_{G-i}=\mathrm{m}_{G-j}=\mathrm{m} \end{array}\right\} \text{ for all } i, j\in G. \qquad (42)$$

Consequently, the optimal solution for each firm will be such that

$$y_i^*=y_j^*=y^*=\tfrac{1}{g}y_G^*. \qquad (43)$$

Since in the equilibrium each firm's expectations for the others behaviors are supposed to be borne out,

$$y_G^*=gy^*=\mathrm{k}+(1+\mathrm{m})y^*. \qquad (44)$$

That is,

$$\mathrm{k}=(g-\mathrm{m}-1)y^*. \qquad (45)$$

Note that the condition $\mathrm{k}\geq 0$ now amounts to the condition

$$\mathrm{m}\leq g-1. \qquad (46)$$

Substituting these values in (39) gives

$$V_{Y_i}=V_{Y_j}=V_Y=(1+m_E)(\beta_Y-r\gamma_Y\eta_Y-r\eta_Y^2k_E)$$
$$-(1+m_E)^2(1+\mathrm{m})r_i\eta_Y^2(g-\mathrm{m}-1)y^*. \qquad (47)$$

Also, define

$$W_Y=\beta_Y-r\gamma_Y\eta_Y=\frac{\partial F}{\partial y} \qquad y=0,\ \mathrm{m}=0,\ m_E=0,\ k_E=0. \qquad (48)$$

Therefore,

$$V_Y=(1+m_E)(1+\mathrm{m})(W_Y-r\eta_Y^2k_E)-(1+m_E)^2((1+\mathrm{m})r_i\eta_Y^2(g-\mathrm{m}-1))y^*. \quad (49)$$

Consequently, (37) can be simplified as

$$y^*=\frac{r\eta_X^2R-V_X+(1+m_E)(1+\mathrm{m})(W_Y-r\eta_Y^2k_E)}{r\eta_X^2+r\eta_Y^2(1+m_E)^2(1+\mathrm{m})g}. \qquad (50)$$

This expression provides the individually rational (or actual) investment in Y on the part of each firm in a homogeneous industry. The total industry's investment will be obtained by the simple formula

$$y_G^*=gy^*. \qquad (51)$$

Of course, this solution will hold only if

$$R_G>gy^*\geq\Pi. \qquad (52)$$

Expression (50) is fairly general and accounts for the effects of government intervention. In the absence of any intervention from the government (i.e., when $k_E=m_E=0$), the industry's total allocation to Y will be given by

$$y_G^*=g\,\frac{r\eta_X^2R-V_X+(1+\mathrm{m})W_Y}{r[\eta_X^2+\eta_Y^2(1+\mathrm{m})g]}. \qquad (53)$$

Pareto Optimal Equilibrium

As discussed in [16], in the homogeneous case the condition for Pareto optimality (that each firm should share the marginal investment in Y in proportion to its marginal benefits)[7] amounts to

$$\mathrm{m}=g-1. \qquad (54)$$

[7]The concept of Pareto optimality is discussed more fully in [16, p. 1005] and [20, pp. 27-31].

Hence the Pareto optimal investment (y_G^{**}) in the absence of government intervention would be given by substituting (54) in (53). That is,

$$y_G^{**} = g\frac{r\eta_X^2 R - V_X + gW_Y}{r(\eta_X^2 + \eta_Y^2 g^2)}. \quad (55)$$

ANALYSIS

Suboptimality in the Absence of Government Intervention

Differentiating y_G^* in (53) with respect to m gives

$$\frac{\partial y_G^*}{\partial \mathrm{m}} = \frac{g}{r}\frac{W_Y\eta_X^2 + \eta_Y^2 g(1+\mathrm{m}) - \eta_Y^2 g r\eta_X^2 R - V_X + (1+\mathrm{m})W_Y}{\eta_X^2 + \eta_Y^2 g(1+\mathrm{m})^2}$$

$$= \frac{g}{r}\frac{\eta_X^2 W_Y + \eta_Y^2 g(V_X - r\eta_X^2 R)}{\eta_X^2 + \eta_Y^2 g(1+\mathrm{m})^2}. \quad (56)$$

But from conditions (20) and (15), respectively, it follows that

$$W_Y \geq 0 \quad (57)$$

and

$$V_X - r\eta_X^2 R \geq 0. \quad (58)$$

Hence, under the stated assumptions

$$\frac{\partial y_G^*}{\partial \mathrm{m}} \geq 0. \quad (59)$$

That is, y_G^* would attain its maximum value when m attains its maximum value. By condition (46) the maximum value of m is $g-1$. But that is also the value of m for Pareto optimality. Thus it is clear that

$$y_G^{**} \geq y_G^*. \quad (60)$$

Since, as discussed earlier, m is likely to be far short of $g-1$ in the absence of government intervention, *the individually rational group investment* (y_G^*) *will be suboptimal* compared to the Pareto optimal group investment (y_G^{**}).

Thus, as in [16], we have established that in a voluntary industry group the individually rational group investment in basic research will be suboptimal in the absence of government intervention. Our analysis contradicts the suggestions of Chamberlin [3], Hardin [8], and Frohlich and Oppenheimer [5] that voluntarily cooperating industry groups may invest socially optimal amounts in inappropriable activities such as basic research. Our analysis supports Olson's [20] position that *such voluntary groups will fail to allocate adequate resources to activities whose* benefits are inappropriable. It strengthens Arrow's [1] conclusion that a free enterprise system will invest suboptimally in basic research, by indicating the truth of that proposition even when firms in an industry are allowed to cooperate with one another on a voluntary basis. Furthermore, the conclusion is true without qualification if all firms in an industry use the Cournot assumption.

However, the mode we present here is not wedded to the Cournot assumption. When observed industry behavior justifies it, m can differ from zero. Clearly, when *perceived industry responses are such that m approaches g−1 little underinvestment* in basic research may be present.

Although the model presented here is not (and is not intended to be) iterative, it can be related to iterative models recently presented by hypergame and metagame theorists [2] [4] [12] [13]. The argument of these theorists can be restated as "over repeated trials or in anticipation of repeated trials of the resource allocation game, the value of m (the perceived and actual matching rate) will go up even in a voluntary group of firms; consequently, underinvestment in inappropriate activities (e.g., *basic research) will be wiped out.*"

As in [16] and [17], the model presented here confirms that if m increases, underinvestment will diminish. However, these models are not aimed at analyzing whether m will or will not change over repeated trials. They simply assume that in any specific short-run resource allocation trial the *perceived* value of m also is its *actual* value in the individually rational equilibrium. This assumption is equivalent to Harsanyi's [9] principal of mutually expected rationality. While our approach clearly represents a simplification, it is a justifiable simplification. Evans and Harris *found it necessary to qualify their conclusion about the long-run optimality* of investment in inappropriable goods by stating that

> In certain circumstances (e.g., epidemics and foreign invasions) any learning lag at all may well be unacceptable. Who among us wishes to allow the loss of life and liberty in order to confirm that voluntary cooperation would be sufficient the second time such irreversible events take place? [4, p. 149]

We believe there are many other circumstances in which society may not want to wait for the second time around.

Even if the metagame theorists are right and the problem of underinvestment in basic research can be resolved through voluntary action in the long run, in the short run there nevertheless may be legitimate desires for government intervention to promote investment in basic research. The questions are what *kinds* of industries would need relatively greater governmental support and what *kinds* of support would be effective and efficient.

We analyze these questions in the remainder of this paper. For that analysis S (the degree of suboptimality of an industry's investment in its basic (inappropriable) research) is defined as

$$S=\frac{y_G^{**}-y_G^*}{y_G^{**}} \tag{61}$$

$$=\frac{(g-\mathrm{m}-1)[\eta_X^2W_Y+g\eta_Y^2(V_X-r\eta_X^2R)]}{(r\eta_X^2R-V_X+gW_Y)(\eta_X^2+\eta_Y^2g(1-\mathrm{m}))} \tag{62}$$

where

$$V_X=\beta_X-r\gamma_X\eta_X \tag{63}$$

and

$$W_Y=\beta_Y-r\gamma_Y\eta_Y. \tag{64}$$

Insensitivity to the Mean of a Firm's or Industry's General Competitive Advantage

In the exponential distribution case [17] we concluded that, other things being equal, the greater the mean competitive advantage of an average firm in an industry (α_X) the smaller the suboptimality of group investment in Y and hence the smaller the need for government intervention to promote that industry's basic research. We also concluded that the greater the general competitive advantage of an industry over other industries (α_Y), the greater the degree of suboptimality of investment in Y and hence the greater the need for government intervention.

Based on the normal distribution of benefits assumed in the present paper, the validity of the conclusions reached in [17] cannot be tested. Instead, it may be noted that since neither α_X nor α_Y is involved in equation (62), the degree of suboptimality of investment in basic research and, consequently, the need for government intervention are not sensitive to the mean competitive advantage of either the firm or the industry as a whole.

Sensitivity to the Standard Deviation of the General Competitive Advantage

On the other hand, since V_X and W_Y do involve γ_X and γ_Y, it seems the *uncertainties associated with general competitive advantages do play a role.* It can be shown that

$$\frac{\partial S}{\partial \gamma_X}=-\frac{(g-\mathrm{m}-1)}{\eta_X^2+\eta_Y^2g(1+\mathrm{m})}\cdot\frac{r\eta_X(\eta_X^2+g^2\eta_Y^2)W_Y}{(r\eta_X^2R-V_X+gW_Y)^2} \tag{65}$$

which is negative under our assumptions. Hence, other things being equal, *the greater the uncertainty associated with the general competitive advantage of a firm, the less the suboptimality of investment in Y.*

This suggests that the finding in [17] should be reinterpreted. In the exponential case, α_X is a measure of both the mean and standard deviation of the general competitive advantage of a firm. Hence, it may be said that there, too, the greater the *uncertainty* associated with the general competitive advantage of an individual firm, the less the suboptimality of investment in Y.

Similarly, since

$$\frac{\partial S}{\partial \gamma_Y}=\frac{(g-\mathrm{m}-1)}{\eta_X^2+\eta_Y^2 g(1+\mathrm{m})}\cdot\frac{r\eta_Y(\eta_X^2+g^2\eta_Y^2)(V_X-r\eta_X^2R)}{(r\eta_X^2R-V_X+gW_Y)^2} \tag{66}$$

is positive under our assumptions, other things being equal, ***the greater the uncertainty associated with the general competitive advantage of the industry as a whole, the greater the degree of suboptimality of investment in Y.***

In short, our investigation of the case of normally distributed benefits here does not invalidate earlier results but suggests a clearer interpretation of those results. What matters is the *uncertainty* associated with the general competitive advantage of a firm or industry rather than the expected value of the competitive advantage.

Sensitivity to Expected Benefits per Dollar of Investment in X and Y

Under our assumptions

$$\frac{\partial S}{\partial \beta_X}=\frac{(g-\mathrm{m}-1)}{[\eta_X^2+\eta_Y^2 g(1+\mathrm{m})]}\cdot\frac{(\eta_X^2+g^2\eta_Y^2)W_Y}{(r\eta_X^2R-V_X+gW_Y)^2} \tag{67}$$

is positive. Hence, as in the exponential case, the larger the expected benefits per dollar of investment in X, the greater the degree of suboptimality of investment in Y. Also, since

$$\frac{\partial S}{\partial \beta_Y}=-\frac{(g-\mathrm{m}-1)}{[\eta_X^2+\eta_Y^2 g(1+\mathrm{m})]}\cdot\frac{\eta_X^2+g^2\eta_Y^2(V_X-r\eta_X^2R)}{(r\eta_X^2R-V_X+gW_Y)^2} \tag{68}$$

is negative, the finding in the exponential case—that the larger the expected benefits per dollar of investment in Y, the smaller the degree of suboptimality of investment in Y—is confirmed.

Sensitivity to Increases in the Standard Deviaton of Benefits *per Dollar of Investment in X and Y*

The algebra involved in establishing the signs of $\partial S/\partial\eta_X$ and $\partial S/\partial\eta_Y$ is rather cumbersome. For the sake of brevity, the following condition for a noncorner solution for y_G^* is used:

$$(1+\mathrm{m})W_Y < V_X + r\eta_Y^2 Rg(1+\mathrm{m}). \tag{69}$$

With that condition, it can be shown that $\partial S/\partial\eta_X$ is negative but $\partial S/\partial\eta_Y$ is positive. That is, other things being equal, an increase in the standard deviation associated with X reduces the suboptimality of investment in Y, and an increase in the standard deviation associated with Y increases the suboptimality of investment in Y. This result would be expected intuitively.

In a sense, in the exponential case we did not have a result for comparison. This is because in the exponential case β_X and β_Y represented increases in both the mean and standard deviation of benefits-per-dollar of investment in X and Y, respectively. In the normal distribution case, β_X and β_Y give the increases in the mean benefits-per-dollar of investment while η_X and η_Y give the increases in the standard deviations of benefits per dollar of investment.

On the other hand, comparing the results here with those in the exponential case, we can say that the degree of suboptimality is more sensitive to the per-dollar increase in the mean expected benefits than to the per-dollar increase in the standard deviation of those benefits. This is an important finding and provides us with a clearer interpretation of the earlier results in [17].

Sensitivity to Investable Resources

It can be shown that

$$\frac{\partial S}{\partial R} = \frac{(g-\mathrm{m}-1)}{[\eta_X^2+\eta_Y^2 g(1+\mathrm{m})]} \cdot \frac{r\eta_X^2(\eta_X^2+g^2\eta_Y^2)W_Y}{(r\eta_X^2 R-V_X+gW_Y)^2} \tag{70}$$

is negative. Hence, other things being equal, the larger the investable resources available to an average firm in the industry, the smaller the degree of suboptimality and the less the need for government intervention to promote basic research in that industry. This is the same result as in the exponential case.

Sensitivity to a Cooperative or Exploitative Climate

Differentiating S with respect to m gives

$$\frac{\partial S}{\partial \mathrm{m}} = \frac{[\eta_X^2 W_Y+g\eta_Y^2(V_X-r\eta_X^2 R)]}{(r\eta_X^2 R-V_X+gW_Y)} \cdot \frac{[\eta_X^2+\eta_Y^2 g(1+\mathrm{m})]+(g-\mathrm{m}-1)\eta_Y^2}{[\eta_X^2+\eta_Y^2 g(1+\mathrm{m})]^2}$$

$$= \frac{[\eta_X^2 W_Y+g\eta_Y^2(V_X-r\eta_X^2 R)]}{(r\eta_X^2 R-V_X+gW_Y)} \cdot \frac{(\eta_X^2+\eta_Y^2 g^2)}{[\eta_X^2+\eta_Y^2 g(1+\mathrm{m})]^2} \tag{71}$$

which is negative under our assumption. Thus, other things being equal, *the greater the value of the perceived matching rate, the less the suboptimality of investment in Y.* Again, this is the same result as in the exponential case [17].

Since a larger value of m indicates a higher degree of perceived cooperation in an industry, our result says that the greater the perceived cooperation, the less the degree of suboptimality of investment in basic research and, consequently, the less the need for government intervention to stimulate basic research in that industry. If, as the metagame theorists believe [4] [12] [13], the value of m increases over repeated trials of the resource allocation game then, according to the result in (71) above, an industry's *investment in its basic research* should approach Pareto optimality as years pass. In any case, changes in the value of m over time is a topic that should be studied empirically rather than theoretically.

Sensitivity to Risk Aversion

It can be shown that

$$\frac{\partial S}{\partial r} = \frac{(g-\mathrm{m}-1)(\eta_X^2+g^2\eta_Y^2)\gamma_Y\eta_Y\beta_X-(\eta_X^2R+\gamma_X\eta_X)\beta_Y}{\eta_X^2+\eta_Y^2 g(1+\mathrm{m})(r\eta_X^2R-V_X+gW_Y)^2}. \tag{72}$$

The sign of (72) depends on the sign of

$$\gamma_Y\eta_Y\beta_X-(\eta_X^2R+\gamma_X\eta_X)\beta_Y. \tag{73}$$

Hence $\partial S/r >, =, < 0$ as

$$\frac{\gamma_Y\eta_Y}{\beta_Y} >, =, < \frac{\gamma_X\eta_X+\eta_X^2R}{\beta_X}. \tag{74}$$

Note that $\frac{\gamma_Y\eta_Y}{\beta_Y}$ is the ratio of the increase in the variation to the increase in the mean of the benefits from Y when the initial expenditure in Y is zero. $\frac{\gamma_X\eta_X+\eta_X^2R}{\beta_X}$ is a similar ratio for the benefits from X when all resources are spent on X. Clearly then, $\frac{\gamma_Y\eta_Y}{\beta_Y}$ is the smallest risk associated with an investment in Y, while $\frac{\gamma_X\eta_X+\eta_X^2R}{\beta_X}$ is the largest risk associated with an investment in X.

As in the exponential case, the result states that if the smallest risk associated with Y is greater than the largest risk associated with X, then as risk aversion increases the degree of suboptimality of investment in Y will increase. However, when the smallest risk associated with Y is smaller than the largest risk associated with X (which is the more likely case), *as the risk aversion increases the degree of suboptimality investment in Y will decrease.* Thus, *the greater the degree of risk aversion displayed by member firms in an industry, the less the suboptimality of that industry's investment in basic research.*

This finding seems to contradict what we might infer from Arrow's [1] work. Arrow did not analyze the relative effects of different levels of risk aversion. However, based on his discussion, we could hypothesize that industries which display greater risk aversion would invest more suboptimally in their basic research than those that display relatively smaller risk aversion. This hypothesis seems to be widely held by many people who study government policy toward R&D. The analysis presented here indicates that this popularly held hypothesis is incorrect. Instead, our analysis finds that greater risk aversion leads to a smaller degree of suboptimality of investment in basic research.

Of course, Arrow's [1] major focus was to identify the variety of factors that could lead to suboptimal investment in R&D on the part of a private enterprise system. The risk associated with R&D activities was one of these factors. Implicit in Arrow's discussion (which considered only the allocation to R&D and not the allocation of resources among R&D and other investment opportunities) was the assumption that investment opportunities other than R&D had relatively low risks associated with them. The model presented here clearly differs. It assumes that both the basic research activity and the alternative investment opportunities have specific risks associated with them. It finds that, in the most likely case, less risk aversion leads to greater suboptimality of investment in basic research.

Note that Arrow's statement—"We expect a free enterprise economy to underinvest in invention and research (as compared with an ideal) because it is risky, because the product can be appropriated only to a limited extent and because of increasing returns in use" [1, p. 619]—leaves the impression that these factors are *additive* as far as their effect on the suboptimality of investment in R&D is concerned.

As in [17], the present analysis takes into account the *joint* effects of inappropriability of R&D activity *and* risk aversion on the part of individual firms. Our analysis suggests that if an industry is concerned with basic research investment opportunities, risk aversion on the part of member firms in fact may help the industry to reduce the degree of its suboptimal investment in basic research.

This is an important finding. Based on these findings we suggest that, contrary to popular belief, a risk-averse industry calls for *less* government intervention in order to encourage its basic research than an industry with less risk aversion.

Sensitivity to Group Size

As in [17], we considered several changes in group size.

Case I: r, R, V_X, W_Y, and m are independent of g. That is, a dynamic group is faced with a joint investment opportunity characterized by inappropriable benefits and nonrivalness.[8]

[8]Economists classify public goods (i.e., goods characterized by inappropriability) into two important categories: those characterized by *rivalness* and those characterized by *nonrivalness*. When the total benefits of a public good to an industry are *fixed*, the benefits that one firm obtains must reduce the benefits other firms can obtain—and the good is said to be characterized by *rivalness*. When the benefits of a public good to one firm are *not* limited by benefits derived by other firms (and the larger the number of firms in the industry, the greater the total benefits of the industry), the good is said to be characterized by *nonrivalness*. See [20, pp. 36-43, 76-89].

In this case when benefits were assumed to be exponentially distributed $\partial S/\partial g$ always was positive. The algebra is more complex when benefits are assumed to be distributed normally. In spite of considerable effort, we were unable to duplicate that result. What we can prove is that any one of the following two conditions is sufficient for $\partial S/\partial g$ to be positive:

$$2(\mathrm{m}+1)>g \tag{75}$$

or

$$\eta_X^2>\eta_Y^2 g^2. \tag{76}$$

On the other hand, these conditions are not necessary for $\partial S/\partial g$ to be positive. In fact, all the numerical examples using normally distributed benefits that we constructed showed that as group size increases, the suboptimality of investment in Y also increases (whether the above conditions are satisfied or not).

Also, from equations (71) and (69) and their related discussions, both $\partial S/\partial \mathrm{m}$ and $\partial S/\partial \eta_X$ are negative. Thus when either of the above conditions is violated (because m or η_X is smaller), the degree of suboptimality ought to be greater than when the conditions are satisfied.

Therefore, here it seems most plausible that, as in the exponential case, $\partial S/\partial g$ always will be positive. However the proof of this result must be left for future research.

Case 2: A dynamic group is faced with a basic research investment opportunity characterized by inappropriable benefits and rivalness. In this case, by definition the group's benefits (B_{YG}) are fixed, regardless of group size. Consequently, here B_{Yi} is an inverse function of g. It follows that α_Y and β_Y are related inversely to g while γ_Y and η_Y are related inversely to $\sqrt{g}$.

Again, the algebra involved is rather cumbersome. We omit it but report that the expression for $\partial S/\partial g$ in this case involves (in the numerator) a few positive terms in addition to terms in the expression for $\partial S/\partial g$ in Case 1 above. Thus, if $\partial S/\partial g$ is positive in Case 1, it must be positive here also.

Case 3: r, V_X, W_Y, and m are independent of g but $R=R_G/g$ is an inverse function of g. In this case, the argument is the same as for Case 2 above. It can be shown that if $\partial S/\partial g$ is positive for Case 1, it must be positive for Case 3 also.

Case 4: In [17] it was reported that if m is an increasing function of g, we cannot arrive at a definitive result for the sign of $\partial S/\partial g$. Therefore, we did not attempt to verify that result here.

In short, as regards sensitivity to group size, a conclusive verification of our results calls for additional work on Case 1 above. However, at present it can be said that the earlier results have not been contradicted.

Evaluation of Federal Policy Instruments

Let us now turn our attention to the evaluation of two fiscal instruments for stimulating basic research: the provision of a matching subsidy (m_E) and the provision of seed money (k_E). Assume the government's concern is to bridge the gap between an industry's actual investment in basic research (y_G^*, as given by (53)) and its group rational investment (y_G^{**}, as given by (55)) in the absence of any government intervention.

With government intervention, the total investment in Y is given by (34). Let us represent that condition by $y_{T|E}^*$, to emphasize the presence of government intervention. Thus, government wants $y_{T|E}^*$ as close to y_G^{**} of (55) as possible. One way of accomplishing this is for the government to spend the desired amount itself. However, we assume the goverment's real objective is to increase industry's investment as much as possible without creating an unwarranted burden in terms of tax dollars. Equation (77) gives the industry's investment in Y under government intervention, which we represent by $y_{G|E}^*$ to emphasize the presence of intervention:

$$y_{G|E}^* = (g)\frac{r\eta_X^2 R - V_X + (1+m_E)(1+\text{m})(W_Y - r\eta_Y^2 k_E)}{r\eta_X^2 + r\eta_Y^2(1+m_E)^2(1+\text{m})g}. \tag{77}$$

The net effect of government intervention on the industry's investment is given by $y_{G|E}^* - y_G^* = N_{G|E}$. An instrument is considered effective only if $N_{G|E}$ is positive.

With two or more effective instruments, concern is focused on the efficiency (e) of the tax dollar under each. The cost of a measure to the government is $C_E = y_{T|E}^*$. The ratio $e = N_{G|E}/C_E$ will measure the efficiency of an instrument.

Effectiveness of Seed Money

Under the assumption that the benefit function, risk aversion, and perceived cooperation in an industry are not affected by the presence of government intervention, differentiating $y_{G|E}^*$ with respect to k_E gives

$$\frac{\partial y_{G|E}^*}{\partial k_E} = \frac{g\eta_Y^2(1+m_E)(1+\text{m})}{\eta_X^2 + \eta_Y^2(1+m_E)^2(1+\text{m})g}. \tag{78}$$

Under our assumptions, this is negative. Thus, as in the exponential case, here also industry's investment in Y will be a decreasing function of the seed money provided by the government. Since $N_{G|E}$ in this case is negative, this implies that the provision of seed money is not an effective measure of government intervention. In fact, the provision of seed money must be considered *counterproductive* since it *reduces* the industry's investment. Intuitively, this occurs because as the government increases its investment in basic research, firms feel freer to allocate their own funds to those activities with more appropriable benefits.

Effectiveness/Efficiency of Matching Subsidies

The algebra involved in the derivative of $y^*_{G|E}$ with respect to m_E is rather cumbersome and is not presented here. Nevertheless, it can be reported that for $\partial y^*_{G|E}/\partial m_E$ to be positive, a sufficient condition is

$$y^*_G \leq \tfrac{1}{2} R_G. \tag{79}$$

Condition (79) states that if, in the absence of government intervention, an industry is expected to invest less than half its total available resources in basic research (a condition that would be valid in almost all real-life situations), then *a matching subsidy is an effective instrument of government policy to stimulate that investment.*

Assume the government is considering using only a matching subsidy (i.e., $k_E=0$, $m_E>0$). The net increase in an industry's investment with a given m_E will be

$$N_{G|E} = y^*_{G|E} - y^*_G \tag{80}$$

while the cost of the instrument to the government will be

$$C_E = m_E y^*_{G|E}. \tag{81}$$

Hence the efficiency of a matching subsidy will be

$$e = \frac{N_{G|E}}{C_E} = \frac{1}{m_E}\left(1 - \frac{y^*_G}{y^*_{G|E}}\right). \tag{82}$$

Thus,

$$e = \frac{1}{m_E}\left\{1 - \frac{r\eta_X^2 R - V_X + (1+m)W_Y}{\eta_X^2 + \eta_Y^2(1+m)g} \cdot \frac{\eta_X^2 + \eta_Y^2(1+m)g(1+m_E)^2}{r\eta_X^2 R - V_X + (1+m)W_Y(1+m_E)}\right\}. \tag{83}$$

As in the exponential distribution case, numerical calculations (with reasonable values for the parameters involved) indicate that the cost efficiency of a matching subsidy often is quite low. In particular, when the government chooses a matching rate that would eliminate suboptimality of investment completely (i.e., such that $y^*_{T|E} = y^{**}_G$), cost efficiency is a small fraction. In other words, at that level the government would spend more than the increase it would induce in the industry's investment in Y. The most important reason for this low efficiency value is that our model assumes a matching subsidy for the entire investment of the industry, including y^*_G (the investment that industry would have made even without the subsidy). This is how a matching subsidy traditionally has been used. The 1981 Reagan tax plan provides a special tax write-off (a form of matching subsidy) for R&D investment that exceeds a firm's average R&D investment during the previous three years. The analysis presented in this paper supports this type of matching subsidy beyond a minimal level (y_G^*). Thus the analysis confirms the finding in the exponential case that a matching subsidy (as traditionally understood) is an effective but not very cost-efficient method of stimulating industry allocation to basic research. It follows, using the logic in [16], that we also can construct an optimal instrument of government intervention when benefits are normally distributed.

Of course, the development and implementation of such an optimal instrument requires that the government have complete information on each of the parameters in the model for every firm in an industry. This is where any discrepancies between a firm's true preferences and its revealed preferences would have to be dealt with. The recent literature on incentive compatibility [6] [10] [19] [21] [22] should be useful in designing such an optimal instrument. In any case, many other measurement problems also must be solved before the optimal instrument can be developed and implemented. For the moment, it can be asserted that this model, along with [16] and [17], yields important insights into the types of industries that require greater (or less) government intervention and the effectiveness/efficiency of certain prevalent methods of intervention (such as the provision of seed money or matching subsidies). Together these models indicate that empirical work on the measurement of various model parameters would be worth pursuing in several industries.

CONCLUSIONS

Several recent works [2] [4] [12] [13] have suggested that, in the long run, the problem of underinvestment in an inappropriable good (e.g., basic research) may disappear because of repeated trials (or the anticipation of repeated trials) of the resource allocation game in an industry. Hirshleifer [11] even argued that, because of "pecuniary" benefits associated with new technical information, a free economy may overinvest in R&D.[9] In view of these works, a blanket conclusion about the suboptimality of investment in basic research is not warranted. At the same time, the possibility in some situations of long-term optimality or short-term overinvestment does not negate observed instances of underinvestment in other situations. Earlier models [16] [17] have aimed at generating policy implications for situations of underinvestment. In this paper, an attempt was made to verify several of the findings in [16] and [17], assuming that a firm's benefits from investments in alternative opportunities are distributed normally rather than exponentially. It is encouraging that most of the earlier findings are confirmed.

[9]Hirshleifer defined "pecuniary" effects as those wealth shifts due to price reevaluations that take place on release and/or utilization of information. These effects are purely redistributive; that is, they may not add much social value. However, for private firms they may be attractive insofar as they lead to the appreciation of assets or stocks for one firm at the expense of other firms. Hirshleifer [11, p. 571] argued that if Eli Whitney (who invented the cotton gin) had speculated on the implications of his invention for the price of cotton and cotton-bearing land, he might have realized benefits far exceeding the value of his patent on the gin. For a few counterarguments, see [16, footnote 2].

Some of the results obtained here suggest more accurate interpretations for the pertinent results in [17]. Because the algebra involved in the case of normally distributed benefits is rather complex, definitive verification of some of our results also must wait for further investigation. Overall, this model has not contradicted the primary findings from earlier work concerning the effectiveness/efficiency of the provision of seed money or matching subsidies. Together these models suggest that the federal government should avoid the use of seed money as a means of stimulating basic research in industry; such expenditures in fact *reduce* the industry's own investment. The models further suggest that matching subsidies be made available only beyond a minimum investment on the part of the industry and that where possible an optimal intervention instrument, involving a flat tax returned through a matching subsidy, be used [16] [17].

The model presented here confirms the following important recommendations made in [16] and [17] for government policy toward the stimulation of basic research in industry. Where feasible, the government should

1. attempt to increase an industry's cooperative spirit and predisposition toward R&D investment (i.e., increase intraindustry matching rates).
2. increase intervention in industries that involve large numbers of firms. Alternatively, it should reduce the number of firms in an industry (at least by *not subsidizing* smaller, marginal firms).
3. tax the benefits from private investments (X_i) more heavily and those from joint investments less heavily.
4. be aware that although industries involving smaller numbers of firms, greater investable resources, greater interindustry competition, less interfirm competition, and greater risk aversion have relatively less need for government intervention, in most industries the degree of suboptimality of investment in inappropriate joint opportunities is fairly high.

It would seem useful to employ the models and methods of analysis developed in these papers ([15], [16], [17], and the present paper) to determine degrees of suboptimality of investment and appropriate government intervention methods. Empirical work toward measuring the various parameters of these models would be worth pursuing in several industries. [Received: June 15, 1984. Accepted: January 22, 1986.]

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