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Notes*
VII

NOTES ON HOFSHI AND KORSH 1972†

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Rami Hofshi and James Korsh [1] (hereafter referred to as H & K) made an important attempt to define a measure of an individual's power in a group. Whereas the direction of their effort is significantly different from earlier efforts in assigning numerical weights to each individual of an organization, there are a number of logical flaws in their model. These comments are meant to identify the inadequacies of the H & K model.

A simple modification of the H & K model is suggested. Deficiencies, not corrected by the suggested modification, clearly demonstrate the need for the development of a totally new model. Nevertheless, it is hoped that these comments do not undermine the merits of the direction taken by H & K, but, in fact, stimulate further efforts founded on concepts that are logically more consistent. It is assumed that the reader is thoroughly familiar with the H & K paper, and, in fact, has a copy of it before him. The comments are simply itemized for convenient organization of this note.

1. Two Logical Flaws

H & K define (p. 53) p_{ij} to be the probability that individual i wins a subjective argument from individual j, given an active difference of opinion occurs between them. v_{ij} is defined as the probability that such an active difference of opinion occurs between i and j. But H & K say that "...the probability is zero that individual i wins a subjective argument from individual j given no active difference of opinion..." (p. 53).

Such a statement is meaningless. If there is no active disagreement, there is no question of winning or losing the argument. Those events simply do not take place since there is agreement to begin with.

H & K also say "...that every individual, in the group, has complete control over himself,...corresponds to $p_{ii}v_{ii} = 1$, for all $i = 1, 2, \ldots, n$" (p. 54). But, $p_{ii}v_{ii} = 1$ implies that $p_{ii} = 1$ and $v_{ii} = 1$. Since both p_{ii}, v_{ii} must always have values between 0 and 1. In words, then, it would imply that an individual i is always in disagreement with himself ($v_{ii} = 1$) and that individual i always wins the argument from himself ($p_{ii} = 1$). But such statements cannot be accepted as the basic assumptions of a theory.

2. The Need for Explicit Recognition of $\bar{v}_{ij}$

H & K do not explicitly recognize any relationship(s) at all between p_{ij} and p_{ji} or between v_{ij} and v_{ji}. Hofshi's original dissertation [2], from which this paper seems to have been developed, provides illustrations in which $v_{ij} = v_{ji}$ and $p_{ij} \leqslant 1 - p_{ji}$. We

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believe these or other appropriate relationships among the variables should be established and used in any model that is built. But more importantly, the flaws mentioned in §1 above arise because of H & K's failure to take into account explicitly the probability of no active disagreement between two individuals.

Let $\bar{v}_{ij}$ $(= \bar{v}_{ji} = 1 - v_{ij})$ be the probability of no active disagreement between individuals i and j. Then, that every individual is in full control of himself corresponds simply (and more logically) to $\bar{v}_{ii} = 1$ for all $i = 1, 2, \ldots, n$.

One can then define an index $b_{ij} = p_{ij}v_{ij} + \bar{v}_{ij}$ as the one to be used in place of the H & K index a_{ij} $(= p_{ij}v_{ij})$.

Observe that, by definition, $b_{ii} = 1$ for all $i = 1, 2, \ldots, n$ and that $0 \leqslant b_{ij} \leqslant 1$ for all i, j. The index b_{ij} becomes particularly relevant when one recognizes, as H & K do, that "Power has to do with one's maximum possible influence or potential ability to influence."

It is this basic premise that prompts H & K to say that "An individual's power increases not only as a larger proportion of group members yield to him in conflict, but also as the relative power of those who have yielded to him increases."

With the explicit definition of $\bar{v}_{ij}$ one would modify this statement to say that "An individual's power increases not only as a larger proportion of group members yield to him in conflict, but also as the relative power of those who have yielded to him, *and those who are in agreement with him from the beginning*, increases."

Thus, if one is working on lines parallel to those of H & K, one would define two indices

$$S_{ir} = \sum_k b_{ik} P_{kr}, \quad S_{jr} = \sum_k b_{jk} P_{kr},$$

where P_{kr} is the power of individual k over individual r, and where P_{ij} increases as S_{ir} increases and P_{ij} decreases as S_{jr} increases. Then P_{ij} can be defined as $P_{ij} = S_{ir}/S_{jr}$.

3. The Modified Model Has Properties Sought by the H & K Model

Under this formulation the property that P_{ij} be nonnegative is satisfied. The second "reasonable" property for P_{ij} that H & K identify (p. 54) would be reworded as "If individual j sometimes wins an argument with individual i (directly or indirectly) and individual i never wins an argument with individual j, even indirectly via other individuals and if individual i always has an active argument with individual j, then individual i's power over j is measured to be zero".

In more mathematical terms our second property can be stated as:

For a pair i, j if

Condition 1. $\bar{v}_{ij} = 1 - v_{ij} = 0$.

Condition 2. $p_{ij} = 0$.

Condition 3. $P_{k,j} = 0$ for all k, such that $P_{ik} > 0$.

Condition 4. Either $P_{ji} > 0$ or there is at least one m such that $P_{mi} > 0$ and $p_{jm}v_{jm} + \bar{v}_{jm} > 0$.

are satisfied, then $P_{ij} = 0$.

The third property (theorem on p. 56) can be restated as "If two individuals, i and j, interact with the same individuals such that $b_{ik} = sb_{jk}$ for $k = 1, 2, \ldots, i-1, i+1, \ldots, j-1, j+1, \ldots, n$, then $P_{ir} = sP_{jr}$ if either $b_{ij} = b_{ji} = 0$ or $b_{ij} = s^2 b_{ji}$."

It is also clear that H & K's discussion on the solutions to simultaneous equations would be fully valid with the simple substitution of the b_{ij}'s for all a_{ij}'s.

Incorporating the probability of no disagreement gets the model closer to the realities of group dynamics. Many times we come across individuals in organizations, who seem to possess their power through the mere ability of being able to guess what position the most powerful member(s) is going to take. Their arguments are won for them by others who simply "happen" to agree with them. These types of persons would have received no weight at all under the H & K original model.

4. The Deficiencies Not Corrected

The foregoing discussion corrects only two of the basic, logical flaws in the H & K model. However, there are two other deficiencies associated with the H & K model (and the revised version presented above).

(a) *Double Counting*

Consider the H & K statement that "$\sum_{k=1}^{n} a_{ik}$ is the expected number of group members who will yield to individual i" (p. 54); under the revised version one would claim that "$\sum_{k=1}^{n} b_{ik}$ is the expected number of group members who will be in agreement with individual i (either from the beginning or having "yielded" to i, directly or indirectly)".

In a group of two individuals, i and j, such that $\bar{v}_{ij} = 0$, $v_{ij} = 1$, $p_{ij} = 0.6$ and $p_{ji} = 0.3$. The expected number of people "yielding to" or "agreeing with" i would be 1.6, and simultaneously the expected number of people "yielding to" or "agreeing with" j would be 1.3, for the total of 2.9.

Thus, in the H & K model (even when modified) there is a serious mistake of "double-counting." The double counting seems to be carried on in the definition of T_{ir}, or S_{ir}. Because of this double counting, for a group of only two individuals (where there is no sequential influence), P_{ij} turns out to be $(b_{ij}/b_{ji})^{1/2}$ rather than the logically more acceptable expression like b_{ij}/b_{ji}.

The example of a group of two persons is rather dramatic. The impact of the double counting may not be very distorting in large groups (for which H & K intended to develop the indices, see p. 52), particularly when there is double counting both in the numerator (S_{ir}) and the denominator (S_{jr}). Thus, though not rigorously accurate, the model may be operational in large groups.

(b) *Infinite Power Values*

It is obvious that in this model $P_{ij} = 1/P_{ji}$. It follows that if P_{ji} can be 0, P_{ij} can be infinite.

But, the primary purpose H & K have in mind for the power indices, so developed, is to use them as weights in finding the group preference structure from individual preference structures. Hofshi's dissertation [2] indicates that they planned to use these indices as below:

If $U_1, U_2, \ldots, U_n$ are the utilities associated, by members $1, 2, \ldots, n$ respectively, with a particular outcome and if $P_{1r}, P_{2r}, \ldots, P_{nr}$ are the power indices of individuals $1, 2, \ldots, n$ with respect to an arbitrarily chosen group member r, then the utility of that outcome for the group as a whole is

$$U_g = \frac{U_1 P_{1r} + U_2 P_{2r} + \cdots + U_n P_{nr}}{P_{1r} + P_{2r} + \cdots + P_{nr}} .$$

It is easy to see that when any one of the P_{ir}'s is infinite the model leads to indeterminate results. Thus, our definition of power may lead to insurmountable problems. It may also be remarked that on the one hand we are prepared to accept power values greater than 1, and on the other hand we say that full control means power = 1. (That is how we say $P_{ii} = 1$.)

Operationally, however, the problem created by the infinite values may be remote in real organizations and may be solved in most cases by the appropriate choice of r.

Thus, both of these deficiencies may allow for the operational use of the modified model in *large organizations*, for applications best elaborated on by Hofshi [2].

5. Concluding Remarks

From the foregoing discussion the need for the development of a totally new model is obvious. In the meanwhile, it may be speculated that the modified H & K model is reasonable for use in large organizations. It is hoped that these comments will stimulate further efforts on the development of measures of power. Such efforts should not ignore two important contributions of Hofshi and Korsh:

(1) Their concept of using quantified power measures for the determination of group preference structures given the individual preference structures.

(2) Their concept of accounting for indirect influence of one person over the other via a third person.

References

1. Hofshi, Rami and Korsh, James, "A Measure of an Individual's Power in a Group," *Management Science*, Vol. 19, No. 1 (September 1972).
2. ———, "A General Mathematical Utility and Decision Making Model," University of Pennsylvania, Ph.D., 1970.

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REPLY TO: "NOTES ON HOFSHI AND KORSH 1972"

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1. Summary

We would like to thank Prafulla Joglekar [4] for his concluding statements in which he indicates, what in his judgement, are the important contributions of Hofshi and Korsh regarding the mathematical development of an individual's power in a group [3]. His efforts to stimulate further work in this area are also appreciated.

Joglekar has presented what he believes to be a number of "logical flaws" in the basic model of [3] and a modification which eliminates some but not all of those flaws.

It is shown below that there are no "logical flaws" in the original model [3]. In fact, the author's model [3] and Joglekar's modification [4] represent two distinct criteria or definitions to the problem of quantifying power. No mathematical flaws exist under either criterion. We would like to point out, however, that the original definition of power [3] was carefully chosen to represent, as closely as possible, well accepted verbal definitions of power in the social sciences [1]. The proposed modification [4] of the model is, in our judgement, contrary to those accepted definitions of power which state that an individual's power is dependent on his ability to influence other individuals to act according to his preference structure and *not* their preference structure [1]. It is not generally accepted that an individual can increase his power in a group simply by agreeing with other powerful members of the group.

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