OPTIMAL PRICE AND ORDER QUANTITY STRATEGIES FOR THE RESELLER OF A PRODUCT WITH PRICE SENSITIVE DEMAND

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ABSTRACT

For over forty years, operations researchers have investigated the relationship between a firm's inventory and pricing decisions by combining the concepts of the economic order quantity theory with those of the price theory. Ever since the earliest works on this topic, researchers have assumed that when a product's unit cost is constant and its demand curve is known and stationary, the reseller of the product would find it optimal to buy a fixed quantity every time he buys and to sell the product at a single price throughout the year. However, when inventory cost considerations are to be combined with the considerations of a price sensitive demand curve, one must keep in mind that, once an order quantity has arrived in the reseller's stock, the inventory on hand is a declining function of time. Consequently, during any inventory cycle, the reseller's inventory carrying costs are also a declining function of time. Thus, it seems appropriate that a reseller should set different selling prices at different points in time during an inventory cycle.

In this paper, we present a model that shows that, when the demand curve is known and stationary, the reseller would be better off if he sets two different prices over two different periods within each inventory cycle. Sensitivity analysis shows that this strategy is particularly profitable when demand is highly price sensitive and the inventory ordering and carrying costs are high. Thus, our model may be particularly useful for resellers of undifferentiated commodities (high price elasticity), resellers of imported products (high ordering costs), and resellers of perishable products (high carrying costs).

INTRODUCTION

When the reseller of a product buys the product at a constant unit cost, incurs a fixed cost per order, stores the product at a constant carrying cost per dollar of inventory per year, and faces a constant demand rate over an infinite horizon, the economic order quantity model tells us that the reseller's optimal strategy is to buy a fixed quantity (EOQ) every time he buys. Ignoring inventory related costs, classical price theory tells us that, when a product's demand is price sensitive but the demand curve is known and stationary, the reseller's optimal strategy is to charge a single price throughout the year. Whitin (1955) combined the concepts from the EOQ theory with the concepts from the price theory to investigate the simultaneous determination of price and order quantity decisions of a reseller. Whitin (1955) assumed that, when all the other assumptions of the EOQ model are valid but demand is price sensitive, with a known and stationary demand curve, a reseller's optimal strategy would be, once again, to buy a fixed quantity every time he buys and to sell it at a single price throughout the inventory cycle.

Assuming a stationary demand curve, Kunreuther and Richard (1971) also sought to build a model for the simultaneous determination of optimal price and order quantity decisions for a reseller. Although Kunreuther and Richard (1971) were perhaps unaware of Whitin's (1955) paper, their model was very similar to Whitin's (1955) model and it was also based on the assumption that a reseller's optimal strategy would be to buy a fixed quantity every time he buys and to sell it at a single price throughout the inventory cycle.

In this paper, we show that when demand is price sensitive, a single selling price throughout an inventory cycle is not optimal. During an inventory cycle, a reseller's inventory level and carrying costs are a declining function of time. When a reseller faces a constant demand regardless of his selling price, there is nothing he can do about these changes in his carrying costs within an inventory cycle. As such, the reseller's optimal strategy is to charge a single price throughout an inventory cycle. However, when his demand is price sensitive, the reseller can adopt a time-dependent pricing strategy to minimize the impact of his time-dependent inventory carrying costs. In theory, the reseller would maximize his annual profit by making his selling price a continuous function of his on-hand inventory. The idea would be to charge a lower selling price when on-hand inventory is large. The lower price would generate higher demand. Consequently, the inventory level as well as inventory carrying costs would decline more rapidly.

Of course, a continuously changing price would not be practical in real life since it would involve price changes from each unit of sale to the next and changes that are in fractions of pennies. On the other hand, it seems reasonable that a reseller could set two or three different selling prices during different time periods within an inventory cycle. Adopting a strategy of only two different prices within an inventory cycle, this paper shows that, when demand is price-sensitive, Whitin's (1955) and Kunreuther and Richard's (1971) assumption of a single price during an inventory cycle leads to suboptimal profits for the reseller.

Over the five decades since Whitin's (1955) work, numerous authors (Tersine and Grasso, 1981; Arcelus and Srinivasan, 1987; Ardalan, 1991; Hall, 1992; Martin, 1994; Arcelus and Srinivasan, 1998; and Abad, 2003) have used Whitin's (1955) and Kunreuther and Richard's (1971) models as foundations to their own models. However, none of these authors have questioned Whitin's (1955) and Kunreuther and Richard's (1971) assumption that the reseller's optimal strategy would be to sell the product at a single price throughout the inventory cycle. Considering a situation of price sensitive demand, Abad (1997 and 2003) finds that, in the case of a temporary sale with a forward buying opportunity, a reseller's optimal strategy is to charge two different prices during the last inventory cycle of the quantity bought on sale. Yet, Abad (1997 and 2003) does not consider a similar strategy in the regular inventory cycle of a product with price sensitive demand.

In this paper, we present a model that shows that, when the demand curve is known and stationary, the reseller is better off if he sets two different prices over two different periods within each inventory cycle. In what follows, first we recapitulate Whitin's (1955) and Kunreuther and Richard's (1971) model. In the next section, we present our own model. Then, we provide several numerical examples considering linear as well as non-linear demand curves and varied values of relevant parameters. The final section provides the conclusions of our analysis along with some directions for future research.

THE WHITIN MODEL

Both Whitin (1955) and Kunreuther and Richard (1971) consider a situation where all the other assumptions of the EOQ model are valid but demand is price sensitive, with a known and stationary demand curve. Although Whitin's (1955) notation is different from Kunreuther and Richard's (1971) notation, and although there are also some minor differences in the details of the two models, the following captures the basics of both the models. For brevity, in the rest of this paper, we shall refer to the model below as the "Whitin model."

Let,
C = The reseller's known and constant unit cost of buying the product
S = The reseller's known and constant ordering cost per order
I = The reseller's carrying costs per dollar of inventory per year
Q = The reseller's order quantity per order.
P = The reseller's selling price per unit
D = Reseller's annual demand at the selling price, P.
D = f(P) = A known, stationary and declining function of P.
D' = f'(P) = The first derivative of D with respect to P.
Z_W = The reseller's annual profit

Then, the reseller's annual profit is given by

$$Z_W = (P - C)D - IC(Q/2) - S(D/Q) \quad (1)$$

This profit function is maximized when

$$Q = [2DS/(IC)]^{1/2} \quad (2)$$

And

$$P = C + [SIC/(2D)]^{1/2} - D/D' \quad (3)$$

By solving for Equations (2) and (3) simultaneously, one can obtain the optimal values of the price and the order quantity.

OUR MODEL

We retain all of the assumptions of the foregoing model except assuming that the reseller sells each unit at a single price throughout an inventory cycle. Instead, we assume that the reseller sets two different prices over two different periods within each inventory cycle.

Let,
The reseller's total order quantity consists of two portions, Q_1 and Q_2, such that $Q = Q_1 + Q_2$.
Then, the reseller's average inventory during the first portion of an inventory cycle is given by $[(Q_1/2) + Q_2]$ and his average inventory during the second portion of an inventory cycle is given by $[Q_2/2]$.
P_1 = The reseller's selling price for the first Q_1 units of each order quantity
P_2 = The reseller's selling price for the remaining Q_2 units of an order quantity
$D_1 = f(P_1)$ = the annual demand rate at price, P_1
$D_2 = f(P_2)$ = the annual demand rate at price, P_2
$T_1 = Q_1/D_1$ = The duration of the first portion of each inventory cycle
$T_2 = Q_2/D_2$ = The duration of the second portion of each inventory cycle
$T = T_1 + T_2 = Q_1/D_1 + Q_2/D_2$ = The total duration of an inventory cycle
Y_0 = The reseller's profit per inventory cycle under our model
Z_0 = The reseller's annual profit under our model

With this notation, in our model, the reseller's per cycle profit is given by

$$Y_O = (P_1 - C)Q_1 - ICT_1[(Q_1/2) + Q_2] + (P_2 - C)Q_2 - IC(Q_2/D_2)[Q_2/2] - S \quad (4)$$

or

$$Y_O = (P_1 - C)Q_1 - IC(Q_1/D_1)[(Q_1/2) + Q_2] + (P_2 - C)Q_2 - IC(Q_2/D_2)[Q_2/2] - S \quad (5)$$

And his annual profit is given by

$$Z_O = Y_O/[(Q_1/D_1) + (Q_2/D_2)] \quad (6)$$

Now, the annual profit is a function of four variables, the two prices and the two quantities sold at those prices. Obtaining the mathematical conditions for the optimality of this function is an involved process. Perhaps, that is the reason why Whitin (1955) and Kunreuther and Richard (1971) assumed a single selling price during the entire inventory cycle. However, in today's computing environment, we can simply use Microsoft Excel's® Solver function to solve our problem. Of course, we need to specify certain constraints, such as the selling prices must each be greater than the reseller's unit cost for the product and that the quantities to be sold at those prices are each non-negative. An added advantage of Microsoft Excel® is that it does not force us to settle for fractional values of our decision variables. Instead, it allows us to impose such realistic constraints as prices must be in integer values of pennies and inventory quantities must also be whole numbers. We use these facilities of Microsoft Excel's® Solver to compute the numerical examples presented in the next section. As those examples show, a reseller who employs our model reaps greater annual profit than the profit realized by a reseller who employs the Whitin model.

NUMERICAL EXAMPLES

In Table 1, we provide several numerical examples comparing our model's assumptions, decision variables, and maximized profits with those of the Whitin model. Table 1 has two major sections. The assumptions section lists the assumed values of the unit cost, the ordering cost, the carrying cost and the demand function. The results section itself consists of two portions. The first portion shows the optimal decisions of the Whitin model and the profit resulting from those decisions. The second portion of the results section shows the optimal decisions of our model and the profit resulting from those decisions. The last column of Table 1 shows the percent increase in profit provided by our model over the profit provided by the Whitin model.

Each row in Table 1 serves as a different numerical example. Consider the first numerical example in the first row. We assume that the reseller's unit cost for a product is $8. His ordering costs are $300 per order. His annual carrying costs are 25% of the inventory value. The reseller's demand function is linear and his demand decreases by 1,000 units for every dollar increase in the selling price. With these assumptions, under the Whitin model, the reseller's optimal selling price is $10.20 and his optimal order quantity is 735 units per order. These policies result in an annual profit of $2,490.31 for the reseller. In contrast, our model recommends that the reseller should order 745 units per order and sell the first 390 units of his stock at $10.10 per unit, but sell the remaining 355 units at $10.31 per unit. Thus, in the early part of an inventory cycle, when his inventory level is higher, the reseller would charge a lower price and move his inventory faster. In the latter part of the inventory cycle, when his inventory level is lower, the reseller would charge a higher price since now he does not need to move his inventory as fast. A reseller using our model would reap an annual profit of $2,500.91, an increase of 0.43% over the profit he would have made using Whitin's model. The numerical example in the first row serves as a base case for situations involving a linear demand. The next six numerical examples, all assuming a linear demand function, consider a change in only one of the assumptions of the base case. In each row, the changed assumption is highlighted. Thus, numerical examples in Rows 2 and 3 retain all the assumptions of the base case except the assumed value of the ordering cost. Note that as the ordering cost increases, compared to the Whitin model, our model provides a larger percent increase for the reseller. Rows 4 and 5 of Table 1 consider a change in the reseller's inventory carrying cost per dollar of inventory. As the inventory carrying cost increases, our model's advantage over the Whitin model increases. Rows 6 and 7 consider a change in the demand elasticity. Once again, as demand elasticity increases, our model's advantage over the Whitin model increases. From the first seven numerical examples in Table 1 it is clear that the increase in profit given by our model is rather modest (less than 1%) in most situations. In Row 8, we provide a numerical example where every one of the parameters is favorable to our model. In such a situation, the use of our model can provide a reseller with a substantial advantage (a 3.5% increase in profit) compared to the use of the Whitin model.

The last eight rows of Table 1 consider a reseller facing a non-linear demand function. Once again, the numerical example in the first one of those 8 rows provides the base case for comparison with the rest of the numerical examples. As can be seen, all of the conclusions of the linear demand curve situations are fully confirmed by the situations of non-linear demand function. There is only one new insight from these numerical examples. A non-linear demand function favors our model more than a linear demand function.

In summary, our sensitivity analysis has shown that, compared to the Whitin model, our model is particularly advantageous when demand is highly price sensitive and the inventory ordering and carrying costs are high. Thus, our model may be particularly useful for resellers of undifferentiated commodities (high price elasticity), resellers of imported products (high ordering costs), and resellers of perishable products (high carrying costs).

CONCLUSION

We have argued that during any inventory cycle, a reseller's inventory level and carrying costs are a declining function of time. When the reseller's demand is price sensitive, the reseller can minimize the impact of high carrying cost during the early portion of an inventory cycle by lowering his selling price and moving his inventory faster. In theory, the reseller would maximize his annual profit by making his selling price a continuous function of his on-hand inventory. However, a continuously changing price is not a practical strategy. Adopting a strategy of only two different prices within an inventory cycle, we have shown that, in situations of price-sensitive demand, Whitin's (1955) and Kunreuther and Richard's (1971) assumption of a single price during an inventory cycle leads to suboptimal profits for the reseller.

We have shown that our model always results in a higher profit for the reseller compared to profit given by the Whitin model. The increase in profit given by our model is rather modest (less than 1%) in most situations. However, this increase in profit can be sizable for resellers who deal in imported items, and/or perishable items, and/or undifferentiated commodities.

Insofar as our model has shown the suboptimality of one of the fundamental assumptions of the body of literature that combines the concepts of inventory theory with those of pricing theory, our model can be extended to re-examine numerous research areas that have embraced that assumption until now. These areas include: a manufacturer's optimal lot size and pricing policy, a reseller's response to a temporary price reduction, and a reseller's price and order quantity strategies when demand is not only a function of price, but also a function of the inventory level.

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