OPTIMAL PRICE AND LOT SIZE IN FACE OF A SUPPLIER'S TEMPORARY PRICE REDUCTION OVER AN INTERVAL

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ABSTRACT

Assuming price sensitive demand, a recent paper presents a model for a reseller's response to a supplier's temporary price reduction over an interval. However, the paper proposes rather complicated computational procedures for obtaining the optimal solutions to the model. We show that effective use of Excel solver can simplify the solution procedure for the practicing mangers. Thus, our work should help improve the likelihood of actual use of the model in practice. Secondly, we identify an inconsistency in the optimal strategies recommended by that model for the two situations of temporary price reductions it addresses. We show that the inconsistency originates in an unnecessarily restrictive assumption. Using a less restrictive assumption, we present a modified model. Our model not only produces internally consistent results, it also yields a higher profit for the reseller. In short, we present a model that is less restrictive, more profitable, and easier for practicing managers to actually use. Finally, based on the analysis in this paper, we identify two facets of some of the current OR modeling efforts that should be avoided in future.

INTRODUCTION

How a reseller should respond to a supplier's temporary price reduction has been a subject of several studies. Most prior works assumed that the price reduction was offered only at one point in time (Abad, 1997; Arcelus & Srinivasan, 1998) and/or that the end demand was not sensitive to the reseller's selling price (Lev & Weiss, 1990; Arcelus & Srinivasan, 1991, Goyal, et al., 1991). In a recent paper, Abad (2003) makes a significant contribution by relaxing these restrictive assumptions.

Assuming price sensitive demand, Abad (2003) presents a model for a reseller's response to a supplier's temporary price reduction over an interval. He considers two types of temporary price reduction situations faced by a reseller:

1. Situation 1. The discount is applicable only to units resold during the promotion. The reseller must resell all the units bought at discount within the promotion period and at a price lower than the reseller's normal price for the product.
2. Situation 2. The discount is applicable on units purchased during the supplier's promotion. It is up to the reseller whether the discounted units are resold during the promotion period or later, and whether the quantity purchased on discount is resold at a reduced price or not.

To maximize the reseller's profit function in each one of the situations he addresses, as is common in operations research, assuming continuous variables, Abad (2003) establishes the first order conditions for optimality. However, since one of his decision variables must be integer valued, he then outlines a complicated procedure (requiring manual iterations) to arrive at the optimal integer value of that variable and the corresponding optimal values of the remaining variables. Real world managers may find that procedure to be too burdensome, if not intimidating. Given that Abad's (2003) recommended procedure involves the use of Excel Solver, we outline a simpler procedure that should enhance the likelihood of use of his model in real life.

In his model addressing Situation 2, Abad (2003) assumes that, towards the end of the promotion period, the reseller would purchase a large lot at a reduced price, sell some part of that large lot at a reduced price, and then sell the rest of the lot at the reseller's regular price. We show that this assumption makes Abad's (2003) results for Situation 2 inconsistent with his results for Situation 1. We present a modified model for Situation 2 by relaxing Abad's assumption that the reseller will sell the last portion of his large lot only at the reseller's regular price. In addition to being consistent with the Situation 1 model, our Situation 2 model provides optimal price and order quantity strategies that are less restrictive and that result in a greater profit for the reseller.

In what follows, we first recapitulate Abad's (2003) model and his numerical example for all the situations he addresses. We show that when its full capabilities are used, Excel Solver can yield a globally optimal solution to Abad's model without resorting to the complicated computational procedure outlined by Abad (2003). Furthermore, Excel Solver allows us to take care of such practical considerations as prices must be in integer pennies and order quantities must be integer numbers. We also identify an inconsistency in the optimal decisions recommended by Abad's (2003) model for the two situations it addresses.

Then, we modify Abad's (2003) model for Situation 2. Our model allows the reseller to charge any price he desires for any portion of the last order quantity bought on sale. We show that this modified model results in strategies that are consistent with the optimal strategies for Situation 1. We show that our model also results in a higher profit for a reseller facing Situation 2.

A RECAPITULATION OF ABAD'S (2003) MODEL

Abad (2003) assumes that demand is a known and stationary function of the reseller's selling price. When the supplier has offered no temporary discount, the situation is considered "a regular situation" Based on earlier works by Whitin (1955) and Kunreuther & Richard (1971), Abad (2003) assumes that, in the regular situation, the reseller would use a single order quantity for all the orders and a single resale price throughout the year. Abad (2003) develops the following model for the reseller's annual profit under the regular situation.

v = Regular per unit price charged by the supplier to the reseller
C = Fixed ordering cost for the reseller/ order
r = Reseller's holding cost per dollar per year
p = Reseller's selling price/ unit
$D(p) = D$ = The known and stationary demand function faced by the reseller.
$D(0) < \infty$, $D(p_u) = 0$ for some $p_u \in (0, \infty)$. $D' = \mathrm{d}D(p)/\mathrm{d}p < 0$ for $p \in (0, p_u)$.
Q = Reseller's order quantity

$W(p, Q)$ = Reseller's annual profit in the regular situation which is a function of the reseller's selling price and regular order quantity.

$$W(p, Q) = [p - v - (C/Q)]D(p) - Qrv/2. \quad (1)$$

In his numerical example, Abad (2003) uses:

v = \$8/unit, C = \$80/order, r = \$0.50/\$/yr, and $D(p) = 10000000p^{-3}$

Although Abad (2003) provides the first order conditions for the maximization of Equation (1), we do not need to recapitulate those. In this paper, we simply use Excel Solver to obtain the optimal solutions to various problems. The use of Excel also allows us to take care of such practical considerations as prices must be in integer pennies and order quantities must be integer valued. Using our procedure and Abad's (2003) values of the parameters, the reseller's optimal order quantity is 466 units/order and his optimal selling price is \$12.26/unit. At this selling price, the reseller's annual demand rate is 5,430 units/yr. These numbers are identical to those reported by Abad. However, the reseller's optimal regular profit as calculated by our procedure (\$21,253.75/year) is slightly lower than that reported by Abad (\$21,574/year). This is primarily due to the realistic constraints we have imposed, namely, prices must be in integer pennies and order quantities must be integer valued.

SITUATION 1

Next, Abad (2003) considers a situation where the supplier has access to the reseller's point-of-sale data and therefore can monitor the selling price and the quantity sold by the reseller under a particular selling price. The assumption is that the reseller qualifies for a discount only on units bought and resold during the promotion period and only if the resale price of those units is smaller than the reseller's regular price. We shall refer to this situation as Situation 1 of supplier's promotion. Based on earlier research in inventory theory, Abad (2003) notes that in this situation, the reseller's optimal strategy is to order and resell one or more integer valued equal size lots during the promotion period. To the notation in the regular situation, Abad (2003) now adds:

p_0 = Reseller's optimal price in the regular situation.
Q_0 = Reseller's optimal order quantity in the regular situation.
W_0 = Reseller's optimal annual profit in the regular situation.
d = The temporary price reduction (\$/unit) offered by the supplier to the reseller
$0 \leq d \leq v$.
T = The duration (in years) over which the supplier has offered a promotion or temporary price reduction.
$T \geq 0$.
m = The number of equal size lots purchased by the reseller during the supplier's temporary promotion.
$m \geq 0$ and integer.
If Q is the quantity ordered in each of the equal size lots, and p is the resale price for that quantity, in Situation 1,

$$Q = D(p)T/m = DT/m. \quad (2)$$

$\pi(p, m)$ = Reseller's incremental profit (i.e., revenue – carrying cost – ordering cost – regular profit) over the promotion period T of Situation 1.

$$\pi(p, m) = (p - v + d)DT - r(v - d)\, D(p)T^2/(2m) - mC - T\, W_0. \quad (3)$$

As is the common practice in operations research, Abad (2003) derives the first order conditions for the maximization of Equation (3) above. Although the optimal price resulting from the simultaneous solution of his first order conditions may be fractional in pennies, Abad (2003) shows no concern for it. However, recognizing that the simultaneous solution of his first order conditions may result in a fractional value for the number of lots purchased, Abad's (2003) recommends a manual iterative procedure to determine the optimal integer value for the number of lots. His recommendation is to follow up the initial solution by using Excel Solver twice, once when the number of lots is fixed at an integer value just below the calculated optimal value, and again when it is fixed at an integer value just above the calculated optimal value. The integer value that results in the higher value of Equation (3) is declared as the optimal number of lots.

As we see it, we do not need to recapitulate Abad's (2003) first order conditions, nor do we need to worry about obtaining fractional values for the number of lots. We simply use Excel Solver to solve for the optimal price and optimal number of lots that maximize equation (3) with the constraint that the number of lots must be integer. Our Solver constraints also specify that all prices be in integer pennies and that the resale price be greater than the reseller's cost per unit but less than the reseller's regular price. To ensure that Solver obtains a globally optimal solution efficiently enough, we modify Solver's default "options" to allow a maximum time of 1,000 seconds, a maximum of 10,000 iterations, a precision level of 0.000000001, and a tolerance of 0.01%. With this approach there is no need for any manual iterations. This simple procedure should make it more likely that Abad's (2003) model is actually used by practicing managers.

Although Abad (2003) does not provide a numerical example Situation 1, in Situation 2 of temporary price reduction, he assumes that the supplier has offered a discount of $0.80/unit over a promotion period of 0.25 year. Using those parameter values in Situation 1 and employing our Excel procedure, the reseller's optimal policy works out as: Order 621 units/order, three times during the promotion period, and resell that order quantity at $11.03/unit. For the reseller, this policy results in an incremental profit of $1,302.41 during the promotion period.

SITUATION 2

Next, Abad (2003) considers what we have labeled as Situation 2 of supplier's promotion. As in Situation 1, in Situation 2 also, the supplier again offers a temporary price reduction over a time interval. However, here the supplier cannot (or does not want to) monitor the reseller's pricing policies and resale timing. As in Situation 1, here also, the reseller's optimal strategy consists of ordering and reselling one or more integer valued equal size lots during the promotion period. In addition, in situation 2, just before the end of the promotion period, the reseller can purchase a large quantity at the discounted price and resell any portion of that quantity at any price the reseller desires at any time after the promotion period.

Unfortunately, Abad (2003) puts an unnecessary constraint on the resale of the large quantity purchased just before the end of Situation 2 promotion. He assumes that the reseller will sell a portion of the large quantity at the exact resale price used in Situation 1 and then sell the remaining portion at only the optimal regular price. To the notation and equations of the regular situation and Situation 1, Abad (2003) now adds the following:

θ = Duration of the first portion the resale of the large lot purchased just before the end of Situation 2 promotion. By Abad's assumption, during this time the reseller maintains the same reduced selling price, p, used during the actual promotion period, T, which required integer number of equal size lots. $\theta \geq 0$.

ψ = Duration of the second portion of the resale of the large lot purchased just before the end of Situation 2 promotion. By Abad's assumption, during this period, the reseller uses the optimal price in the regular situation, p_0. $\psi \geq 0$.

$\pi(p, m, \theta, \psi)$ = Reseller's incremental profit (i.e., revenue – carrying cost – ordering cost – regular profit) from all the inventory purchases made during Situation 2 promotion.

$$\pi(p, m, \theta, \psi) = (p - v + d)D(T + \theta) + (p_0 - v + d)D_0\psi - (m + 1)C - r(v - d)(DT^2/(2m) + D\theta^2/2 + D\psi^2/2 + \psi D_0\theta) - (T + \theta + \psi)W_0. \quad (4)$$

Clearly, this situation has four decision variables: the discounted resale price, the number of equal size lots, and the durations of the first and the second portion of the large lot inventory cycle. Abad (2003) derives the first order conditions for the maximization of Equation (4) and puts forward several propositions. Based on those propositions, he outlines a complicated manual iterative procedure involving four steps to solve the problem. Even then, he warns that his procedure can guarantee only a locally optimal solution. For a globally optimal solution, he recommends trying several different starting values of p. We believe that practicing managers would find Abad's (2003) procedure too burdensome, if not intimidating. However, his procedure is not really necessary if one uses full capabilities of Excel Solver.

We simply use Excel Solver to obtain the optimal solution by maximizing Equation (4) with respect to the four decision variables with appropriate constraints on the acceptable values of the decision variables. As before, to ensure that Solver obtains a globally optimal solution efficiently enough, we modify Solver's "options" to allow a maximum time of 1,000 seconds, a maximum of 10,000 iterations, a precision level of 0.000000001, and a tolerance of 0.01%. The results obtained below show that our simple procedure works.

Using Abad's (2003) parameter values, namely, a discount of \$0.80/unit and the promotion period of 0.25 year, the final solution works out to be a resale price of \$11.10/unit during the 3 identical inventory cycles and the 0.156 year first portion of the large lot cycle, followed by 0.162 year of the second part of the large lot inventory cycle when the resale price is same as the regular optimal price of \$12.26/unit. This means that three times during the sales promotion period, the reseller should order 609 units/order and sell them at \$11.10/unit. Just before the end of the promotion period, he should order 2,017 units, sell 1,139 of those units at \$11.10/unit, and sell the remaining 878 units at the regular price of \$12.26/unit. This solution is the same as the one obtained by Abad (2003) for this situation. The only thing that is different is that while Abad (2003) reports an incremental profit of \$2,289.60 for this situation, our calculations show a profit of only \$2,289.30. This difference is due to the additional practical constraint we have imposed, namely that selling prices must be integer valued in pennies.

So far, we have recapitulated Abad's (2003) model for the two situations of temporary discount he has envisioned. We have shown that, if one uses Excel Solver's full capabilities, one does not really need the complicated computational procedures proposed by Abad (2003) to solve the optimization problems of these situations. The simpler computational procedures we have outlined should make the actual use of Abad's (2003) model by real life resellers more likely.

We find it curious that, even though the supplier's discount amount and the promotion period duration are identical for the numerical examples for both Situation 1 and 2, the optimal price and order quantity decisions provided by Abad's (2003) model for the identical inventory cycles in those two situations are different. For example, in Situation 1, the optimal order quantity for the identical inventory cycles is 621 units/order and the optimal resale price is \$11.03/unit. Whereas in Situation 2, the optimal order quantity for the identical inventory cycles is 609 units/order and the optimal resale price is \$11.10/unit.

We believe the root of this inconsistency lies in the fact that, in dealing with the Situation 2, Abad (2003) has made an unnecessarily restrictive assumption, namely that the identical inventory cycles will be followed by a large order quantity, a portion of which is sold at the same price used during the identical inventory cycles and the remaining quantity is sold at the reseller's regular price. In the next section, we modify Abad's (2003) model to relax this assumption.

A MODIFIED MODEL FOR RESELLER'S RESPONSE IN SITUATION 2

As we have pointed out before, in Situation 2 of a supplier's promotion, the reseller is free to set any selling price for any portion of the quantity bought on discount. As in Abad's (2003) model, we assume that there will be one or more integer valued identical cycles during the promotion period in which the reseller orders a quantity and sells it at a reduced resale price. At the end of those identical cycles and just before the promotion period expires, the reseller orders a large quantity. We also assume that the retailer will resell some portion of that large order quantity at one price and the remaining portion at another price. However, unlike Abad (2003), we do not assume that the first portion must be resold at the same price as the resale price used in the identical inventory cycles, nor do we assume that the second portion must be sold at the reseller's optimal regular price. As we see it, since the supplier cannot (or does not want to) monitor the timing or the price of the resale, the reseller is free to choose any price for any portion of his large order quantity.

To develop our modified model, let us add the following notation to the notation already defined.

Q_1 = Order quantity for the equal size lots during the promotion period.
p_1 = The resale price for each one of the units in Q_1.
Q_2+Q_3 = The total size of the large lot ordered just before end of the promotion period, to be sold over the duration $\theta+\psi$.
Q_2 = The quantity to be resold during time span $[T, T+\theta]$ at price p_2.
Q_3 = The quantity to be resold during time span $[T+\theta, T+\theta+\psi]$ at price p_3.
If D_1, D_2, and D_3, represent the reseller's annual demand rate at the prices p_1, p_2, and p_3, respectively, the following conditions hold:

$$Q_1 = D_1 T/m. \qquad Q_2 = D_2\theta. \qquad Q_3 = D_3\psi. \tag{5}$$

$\pi(p_1, p_2, p_3, m, \theta, \psi)$ = Reseller's incremental profit from all the inventory purchased during Situation 2 promotion

$$\pi(p_1, p_2, p_3, m, \theta, \psi) = (p_1 - v + d)D_1T + (p_2 - v + d)D_2\theta + (p_3 - v + d)D_3\psi - (m+1)C - r(v-d)((D_1T^2/(2m)) + (D_2\theta^2/2) + (D_3\psi^2/2) + D_3\theta\psi) - (T+\theta+\psi)W_0. \tag{6}$$

Note that in our model, there are six decision variables. A mathematically elegant solution to this model would be far more complex than that for Abad's (2003) model involving only four decision variables. Perhaps this is precisely why Abad (2003) used the restrictive assumptions he used. However, given an input of appropriate constraints on the acceptable values of these decision variables, Excel Solver can obtain the optimal solution to the problem. To ensure that Solver obtains a globally optimal solution efficiently enough, we modify Solver's "options" to allow a maximum time of 1,000 seconds, a maximum of 10,000 iterations, and seek a precision level of 0.000000001. Now, for the same numerical values of the parameters used by Abad (2003), we obtain the following solution: We obtain 3 identical inventory cycles with an order quantity of 621 units/order to be resold at $11.03/unit. Note that this is exactly the solution to Situation 1 problem. Thus, with our model, the solutions to the two types of temporary sale situations are mutually consistent.

For the large order just before the end of the temporary discount period, we obtain a total order quantity of 2005 units. Of those, 1,060 units should be sold at $11.20/unit over the first 0.149 year period after the promotion ends and ?945 units should be sold at $12.05/unit over another 0.165 year period. The reseller's optimal incremental profit from all the inventory purchased during the Situation 2 promotion is $2,294.25. Notice that this profit is higher, albeit modestly, than the profit produced by Abad's (2003) model.

CONCLUSION

Assuming price sensitive demand, Abad (2003) presented a model for a reseller's response to a supplier's temporary price reduction over an interval. Even though he used Excel Solver, Abad (2003) proposed rather complicated computational procedures to arrive at the optimal solutions to his model. Abad's (2003) computational procedures may be too intimidating for many managers. As such, they may be reluctant to use his model. We have shown that if one uses Excel Solver's full capabilities, one does not need such complex computational procedures. The simpler computational procedures outlined here should make it more likely that Abad's (2003) model will be actually used by practicing managers.

Second, there was an inconsistency in the optimal strategies recommended by Abad's (2003) model for the two situations of temporary price reductions he addressed. We have shown that the inconsistency results from an unnecessarily restrictive assumption. Using a less restrictive assumption, we presented a modified model. Our model not only produced internally consistent results, it also yielded a higher profit for the reseller. Although that increase in the profit is modest under the parameters values considered, it could be significant in situations of highly elastic demand and sizable inventory ordering and carrying costs. What is more important is that we now have a less restrictive model for a reseller who wants to respond to temporary price reductions from his supplier. Thus, our modification has added some value to Abad's (2003) model.

Finally, based on this analysis we want to observe two facets of some of the current modeling efforts in operations research. As Abad (2003) has done, even when they know that a simple closed form solution for a model cannot be obtained (perhaps because of a requirement that one or more of the decision variables must be integer valued), many OR authors often investigate the first and second order conditions for the optimality of their models before turning to Excel Solver to actually obtain the solution. It is likely that journal editors equate the quality of a manuscript to the existence of that mathematical exercise. We believe that most of these exercises amount to nothing more rediscovery of the wheel and are unnecessary. Secondly, as Abad (2003) has done, many OR authors impose simplifying restrictions on a modeling situation simply to keep the number of decision variables manageable for the mathematical exercise they want to engage in. In the process, sometimes they distort the true properties of a situation, and consequently, obtain a suboptimal solution. Our analysis suggests that these modeling practices should be avoided in future.

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