RESPONDING TO A ONE -TIME -ONLY SALE (OTOS) OF A PRODUCT SUBJECT TO SUDDEN OBSOLESCENCE

Prafulla Joglekar, La Salle University
Patrick Lee, Fairfield University

ABSTRACT

With advancing technologies and shrinking life cycles, today many products are subject to sudden obsolescence. Manufacturers and vendors of products that are subject to sudden obsolescence often announce a one-time-only discount on these products. In this paper, we study a retailer's optimal response to such one-time-only sales (OTOS) of products subject to sudden obsolescence. We build a comprehensive model based on two relevant bodies of literature: the literature on one-time-only sales of non-perishable, non-obsolescent products, and the literature on inventory and pricing decisions for obsolescent products in the absence of any one-time only considerations.

Our model allows for price elasticity, accounts for a various types of inventory holding costs, and deals with obsolescence costs and capital costs separately from the holding costs. Our model also allows for the ordering cost of the special one-time only order to be different from the retailer's regular ordering cost. The model is general enough to accommodate non-obsolescent as well as obsolescent products in situations that do or do not involve an OTOS. A numerical example shows that the use of our model can provide some long-term gain and a particularly attractive short-term improvement in a retailer's profit. Sensitivity analysis shows that the benefits of our model are greatest when the discount is sizable; demand is highly price sensitive; and the retailer's ordering cost for the special order is small.

INTRODUCTION

With rapid advances in technology, abrupt changes in global political situations, and instantaneous dissemination of information in the worldwide market, today product life cycles have decreased dramatically, and a number of products are at risk of becoming obsolete overnight. Swiss watches, computer chips, world maps, breast implants, and Milli Vanilli records are some of the classic examples of this phenomenon. This phenomenon also affects a large number of products whose designs were historically stable for many years, if not decades. For example, fuzzy logic chips have shrunk the lifecycles of such products as washing machines and today's ergonomic focus has rendered obsolescence on older designs of office furniture.

For prudent inventory and pricing decisions on products subject to sudden obsolescence (hereafter called S-Obs products), a retailer must account for the costs of obsolescence carefully. Traditionally, obsolescence costs were treated as a component of the holding costs in the economic order quantity (EOQ) model (Hadley & Whitin, 1963; Naddor, 1966; Silver & Peterson, 1985). Then, some authors dealt with obsolescence costs separately from other inventory carrying costs (Barbosa & Friedman, 1979; Brown, 1982; Hill, Girard & Mabert, 1989). However, these early works were focused on *gradual* rather than *sudden* obsolescence.

Masters (1991) defined *sudden obsolescence* as a situation when a product's lifetime is negative exponentially distributed, and consequently, the probability of obsolescence is constant at any time. Using an *approximate* model, Masters (1991) concluded that for S-Obs products, the use of the EOQ model was appropriate, provided that the obsolescence component was computed as the reciprocal of the product's expected life. Joglekar and Lee (1993, 1996) pointed out that the then current industry practice of estimating annual obsolescence costs at 1 to 3% of a product's cost represented a serious underestimate of the true cost. By Master's (1991) formula even a 3% obsolescence cost implies an expected life of 33 years! Masters (1991) warned that in cases of short-life products, failure to use the proper formula could lead to costs that were five to forty percent higher than the optimal costs.

Masters' (1991) model was an *approximate* one. Using an *exact* formulation, Joglekar and Lee (1993) showed that Masters' model also underestimated the true lifetime costs of his optimal policy while overestimating the optimal order quantity. The associated errors were substantial particularly in the cases of S-Obs products with very short expected lives. Joglekar and Lee (1996) developed a profit maximization model to determine a retailer's optimal order quantity in the face of a manufacturer's quantity discount for S-Obs products. Unlike cost minimization models, a profit maximization model also warns a retailer not to stock a product at all when such a stock results in loss for the retailer.

Not too different from situations of quantity discounts are the commonly observed situations of one-time-only sales (OTOS) of many products. An OTOS occurs because a manufacturer/wholesaler wants to reduce some excess inventory caused by factors such as incorrect forecasts, deliberate excess production, or the threat of impending obsolescence. OTOS allow manufacturers to pass on reduced raw material costs to the reseller, to meet short-term sales goals, to maximize capacity utilization, and/or to add excitement to otherwise mature and mundane products (Abad, 2003). Aucamp and Kuzdrall (1986) have also observed that the situation of an announced permanent price increase, with one last opportunity to buy before that price increase, is mathematically equivalent to an OTOS. Fashion clothes, pop music, and trendy toys are examples of S-Obs products where at the time of a product-introduction, a manufacturer often offers a substantial one-time-only discount to the retailer. Yet, available literature has not studied a retailer's optimal response to such OTOS offers for S-Obs products.

On the other hand, how a retailer should respond to an OTOS of a *non-perishable, non-obsolescent* product has been studied by a number of authors over the last three decades. Using standard EOQ assumptions, earlier works (Naddor, 1966; Brown, 1967; Tersine & Grasso, 1978; Taylor & Bradley, 1985; Lev & Weiss, 1990) developed prescriptive models for determining an optimal special order quantity for a retailer in a variety of OTOS situations. These works assumed a constant demand. Considering price-elasticity, Ardalan (1994; 1995) suggested that, in OTOS situations, in addition to using a special order quantity, a retailer could increase his demand and profits by charging a lower retail price for it. Ardalan (1994) also focused on maximizing the net present value (NPV) of a retailer's cash flows rather than maximizing the per-period profit.

In this paper, we combine Joglekar and Lee's (1996) methodology of analyzing order quantity decisions pertaining to S-Obs products with Ardalan's (1994) approach of simultaneously determining the special price and the special order quantity in the face of an OTOS. In what follows, we establish our notation and develop a model for a retailer's optimal price and order quantity decisions for a price-sensitive S-Obs product in the regular situation, i.e., in the absence of an OTOS. Our model allows for price elasticity, accounts for various types of inventory holding costs, and deals with obsolescence costs and capital costs separately. The model is general enough so that it can be used for obsolescent as well as non-obsolescent products.

Next, we extend the regular situation model to accommodate an OTOS situation. Unlike other available models, we do not assume that a reseller's cost of ordering the special quantity in an OTOS situation will be the same as his regular cost of ordering. We believe that decision-making under special circumstances requires a new model, additional data and greater analytical effort. Hence, the cost of ordering the special quantity is often substantially greater than the regular ordering cost. In order to obtain an accurate estimate of the net advantage of our model's optimal decisions, we use a comparison of the lifetime NPV of the no OTOS situation with the lifetime NPV of a situation involving an OTOS. To gain a clearer perspective on the long term and short-term significance of the net advantage, we look at the net advantage as both, percentage of lifetime NPV and percentage of one cycle NPV.

A numerical example, along with a fairly exhaustive sensitivity analysis, is provided. The numerical example shows that, in many OTOS situations, the use of our model can provide some long-term gain and a particularly attractive short-term improvement in the retailer's NPV. Our analysis also identifies situations where the retailer may be better off not accepting the OTOS discount. The final section provides the conclusion along with some directions for further work.

THE MODEL

Consider a retailer dealing in an S-Obs product characterized by a price sensitive demand function that is time-invariant until obsolescence. Product obsolescence occurs abruptly and completely at a random point in time, which is negative exponentially distributed. At obsolescence, the product is disposed off at a salvage value. Other than these characteristics, standard EOQ assumptions, such as known and constant ordering and carrying costs, zero lead-time, and no stockouts are applicable. In a "regular" situation, i.e., in the absence of an OTOS, the retailer seeks to maximize the NPV of his lifetime cash flows by determining the optimal order quantity and the optimal selling price. Similarly, when faced with an OTOS, a retailer seeks to maximize his lifetime NPV from the special price and order quantity of the OTOS followed by all regular cycles for the rest of the product's life. To evaluate the exact advantage of the optimal OTOS decisions, we look at the difference between these two NPVs.

Throughout this paper, we use the following notation.

A = a constant for the demand function, representing the theoretical maximum demand at zero price

c = retailer's regular unit cost

C_r = retailer's regular ordering cost per order

C_s = ordering cost per order during special cycle

Note: We believe that the commonly used assumption $C_s = C_r$ is unrealistic. C_s is likely to be substantially greater than C_r for three reasons: (i) OTOS policy determination requires the use of a different model, (ii) The OTOS order quantity is likely to be several times the regular order quantity, and (iii) C_s must also include costs of announcing the special retail price, P_s to the retailer's customers.

d = the OTOS discount per unit

H = annual holding costs (such as storage space, and material inspection and handling costs) that are fixed per unit, regardless of the unit cost of the product.

h = annual holding costs (such as deterioration, damage, and pilferage costs) that are fixed per dollar of inventory, but that vary in per unit terms with the unit cost of the product.

Note: Most inventory models assume all holding costs to be of either the H type or of the h type. In real life,one encounters both types. Note also that we deal with the obsolescence costs and the capital costs explicitly and separately. Consequently, neither H nor h includes them.

H_r = total annual holding costs (all except the obsolescence cost) per unit of regular purchase $H_r = [H + (h + i)c]$.

H_s = total annual holding costs (all except the obsolescence cost) per unit of special OTOS purchase $H_s = [H + (h + i)(c - d)]$.

i = cost of capital per dollar per year (used as both, the cost of capital factor in the inventory holding cost and the discount rate for NPV calculations).

k_r = probability, at the beginning of a regular inventory cycle of Qr/Rr years, that the product does not become obsolete during the cycle. $k_r = e^{-Qr/(RrL)}$ (Joglekar & Lee, 1993, 288).

k_s = probability, at the beginning of the special OTOS inventory cycle of Qs/Rs years, that the product does not become obsolete during the cycle. $k_s = e^{-Qs/(RsL)}$ (Joglekar & Lee, 1993, 288).

L = expected life of the product in years

N_r = NPV factor for a cashflow occurring at the end of a regular inventory cycle. $N_r = e^{-iQr/Rr}$.

N_s = NPV factor for a cashflow occurring at the end of the special OTOS inventory cycle. $N_s = e^{-iQs/Rs}$.

P_r = selling price per unit during regular cycle prior to obsolescence

P_s = selling price per unit during special cycle prior to obsolescence

Q_r = order quantity per order during regular cycle

Q_s = order quantity for the OTOS special order

R_r = demand per year during regular cycle, given by the function $R_r = A - \varepsilon P_r$.

R_s = demand per year during special cycle, given by the function $R_s = A - \varepsilon P_s$.

S_o = salvage value per unit after obsolescence, $S_o < (c - d)$.
t = the time at which the product becomes obsolete
$\Delta\pi_L$ = the difference between the expected lifetime profit resulting from the special OTOS P_s and Q_s policies followed by all regular cycles and the expected lifetime profit of all regular cycles in the absence of an OTOS. $\Delta\pi_L = \pi_{Ls} - \pi_{Lr}$
ε = price-elasticity constant of the demand function
π_{cr} = expected profit, in NPV terms, from the first regular inventory cycle
π_{Lr} = expected lifetime profit, in NPV terms, from all regular cycles
π_{cs} = expected profit, in NPV terms, from the special OTOS cycle
π_{Ls} = expected lifetime profit, in NPV terms, from the special OTOS cycle followed by all regular cycles

THE REGULAR SITUATION

The following costs and benefits are in terms of a retailer's expected NPV of lifetime cash flows as expected at the beginning of an inventory cycle. First, the ordering costs and the inventory purchase costs are incurred, representing an NPV of

$$C_T + Q_r c \qquad (1)$$

Product revenues and holding costs depend upon whether and when the product becomes obsolete during an inventory cycle. If the product does not become obsolete, the entire order quantity is sold at the regular price. Subtracting the relevant inventory holding costs from these revenues, the corresponding expected NPV is given by:

$$\int_{Q_r/R_r}^{\infty} [\int_0^{Q_r/R_r} \{R_r P_r - (Q_r - xR_r)[H + (h+i)c]\} e^{-ix} dx](1/L) e^{-t/L} dt$$

Integrating and simplifying this expression by using several equalities established in the notation section, we get

$$P_r R_r (1 - N_r) + H_r [R_r N_r / i + R_r / i - Q_r]\} k_r / i \qquad (2)$$

If obsolescence occurs during the cycle, then the corresponding expected profit contribution is:

$$\int_0^{Q_r/R_r} [\int_0^{t} \{R_r P_r - (Q_r - xR_r)[H + (h+i)c]\} e^{-ix} dx + (Q_r - tR_r) S_o e^{-it}](1/L) e^{-t/L} dt$$

Integrating and simplifying, this expression can be written as

$$[P_rR_r - Q_rh_r + R_rH_r/i]\,[1 - k_r - 1/(1+iL)]/i + [Q_rS_o + P_rR_rN_rk_r/i + R_rH_rN_rk_r/i^2]/(1 + iL)$$
$$- [R_rL/(1 + iL)^2]\,[H_r + S_o]\,[1 - N_rk_r] \qquad (3)$$

The NPV of the profit provided by a regular cycle, as expected at the beginning of that cycle, is given by

$$\begin{aligned}\pi_{cr} &= (2) + (3) - (1)\\ &= Q_rS_o/(1 + iL) + P_rR_rN_rk_r\,[1/(1 + iL) - 1]/i + [R_rH_rN_rk_r]\,[1 + 1/(1 + iL)]/i^2\\ &+ [P_rR_r - Q_rH_r + R_rH_r/i]\,[1 - 1/(1 + iL)]/i\\ &- [R_rL/(1 + iL)^2]\,[H_r + S_o]\,[1 - N_rk_r] - (C_r + Q_rc) \qquad (4)\end{aligned}$$

Given a constant obsolescence rate, and a time-invariant order quantity, the NPV of the product's *lifetime* profit as expected at the beginning of an order cycle is identical to that expected at the beginning of the next order cycle. Hence,

$$\pi_{Lr} = \pi_{cr} + k_r\pi_{Lr}N_r \qquad (5)$$

This can be simplified as

$$\pi_{Lr} = \pi_{cr}/(1 - N_r\,k_r) \qquad (6)$$

The retailer wants to determine his price and order quantity of the regular cycle so as to maximize expected lifetime NPV from all regular cycles. From Joglekar and Lee (1996) we know that this problem is best solved numerically by using the solver function of software such as Excel . Hence, we do not pursue any further manipulation of Equation (6) for a closed form optimization. In our numerical examples, we simply use Excel 's solver function.

THE OTOS SITUATION

We assume that the OTOS discount is available at the beginning of what would have been a regular inventory cycle. Given the discount, the question is whether the retailer should take the discount, and if he does, what special order quantity, he should use, and at what special price he should sell that quantity. We develop a model for calculating expected lifetime NPV of using the special ordering quantity and price at the OTOS followed by regular policies until the end of the product's life. When an inventory cycle of a special order quantity begins, the corresponding ordering costs and the costs of the goods are given by

$$C_s + Q_s(c - d) \qquad (7)$$

Product revenues and holding costs depend upon whether and when the product becomes obsolete during an OTOS cycle. If the product does not become obsolete during the OTOS cycle, all order quantity units of the special cycle will sell at the special price. Subtracting the relevant inventory holding costs from these revenues, we obtain the corresponding NPV by the expression:

$$\int_{Qs/Rs}^{\infty} [\int_{0}^{Qs/Rs} \{R_s P_s - (Q_s - xR_s)[H + (h+i)(c-d)]\} e^{-ix} \mathrm{d}x](1/L) e^{-t/L} \mathrm{d}t$$

Integrating and simplifying, this expression can be written as

$$\{P_s R_s (1 - N_s) + H_s [R_s N_s / i + R_s / i - Q_s]\} k_s / i \qquad (8)$$

If obsolescence occurs during the cycle, the corresponding expected profit contribution, in NPV terms, is given by:

$$\int_{0}^{Qs/Rs} [\int_{0}^{t} \{R_s P_s - (Q_s - xR_s)[H + (h+i)(c+d)]\} e^{-ix} \mathrm{d}x + (Q_s - tR_s) S_o e^{-it}](1/L) e^{-t/L} \mathrm{d}t$$

Integrating and simplifying, this can be expressed as

$$[P_s R_s / i - Q_s H_s / i + R_s H_s / i^2] [1 - k_s - 1/(1 + iL)] + [Q_s S_o + P_s R_s N_s k_s / i + R_s H_s N_s k_s / i^2]/(1 + iL)$$
$$- [R_s L/(1 + iL)^2] [H_s + S_o] [1 - N_s k_s] \qquad (9)$$

Thus, the NPV of all the cashflows of the OTOS cycle is given by

$$\pi_{cs} = (8) + (9) - (7)$$
$$= Q_s S_o/(1 + iL) + P_s R_s N_s k_s / i \, [1/(1 + iL) - 1] + R_s H_s N_s k_s \, [1 + 1/(1 + iL)^2]/i^2$$
$$+ [P_s R_s / i - Q_s H_s / i + R_s H_s / i^2] [1 - 1/(1 + iL)]$$
$$- [R_s L/(1 + iL)^2] [H_s + S_o] [1 - N_s k_s] - [C_s + Q_s (c - d)] \qquad (10)$$

While it is tempting to compare this special cycle NPV with the regular cycle NPV, these NPVs are not comparable since the two cycles involve two different time durations. To determine whether the special order quantity and price policies are more desirable, one must compare the *lifetime* NPV of those special policies followed by regular policies with the expected *lifetime* NPV of using only regular policies throughout.

Expected lifetime NPV from all regular cycles has already been modeled in Equation (6). The expected lifetime NPV of the special OTOS cycle followed by all regular cycles is given by:

$$\pi_{Ls} = \pi_{cs} + \pi_{Lr} k_s N_s \qquad (11)$$

Thus, in the OTOS situation, the retailer wants to determine his special price and order quantity so as to maximize Equation (11). As in the case of the regular situation, we use Excel® 's solver function to solve this problem.

Once the optimized values of the special OTOS cycle are established, the net NPV advantage of the special OTOS policies is given by:

$$\Delta\pi_L = \pi_{Ls} - \pi_{Lr} \qquad (12)$$

If the net advantage is negative, the retailer would reject the OTOS discount. Only if net advantage is positive, the optimal price and order quantity values will be implemented. In that case, for a long-term perspective, we examine the net advantage as a percent of the lifetime NPV with all regular policies. For a short-term perspective, we examine the net advantage as a percent of the NPV resulting from the first regular inventory cycle. It is these values that provide the proper perspective on both the long-term and the short-term gains associated with the use of our model in an OTOS situation. As we see it, while a long-term positive gain is important, the relative magnitude of the short-term gain is the most important consideration. After all, by definition, an OTOS is a one time, short-run deal.

NUMERICAL EXAMPLE

Consider a product with the following parameters:

c = \$10/unit C_r = \$100/order H = \$1/unit/year L = 1 year h = 5%/year
i = 12%/year S_o = \$2/unit R_r = 100,000 - 6,000P_r A = 100,000 units/year
ε = 6,000 units/\$.

We believe these parameter values are fairly realistic. The unit cost and the demand constant are arbitrary and may be different from situation to situation. An ordering cost of \$100/order is within a range of values observed in real life. Together, the assumed holding costs (both fixed and variable) and the assumed cost of capital, result in an assumption of an annual inventory cost (excluding the cost of obsolescence) of 27% of the value of inventory. This is also well within the observed range of values in real life. Because we are focusing on S-Obs products, we assume a salvage value of only 20% of the unit cost and we assume an expected life of only 1 year. We consider a price-elasticity constant implying a reduction 6,000 units in demand for every dollar increase in price. We believe this is also within typically observed range of price-elasticity values.

Table 1

Assumptions, Decisions, and Lifetime Profits of Regular and OTOS Situations

Assumptions:

c = \$10/unit, C_r = \$100/order, H = \$1/unit/year, h = 5%/\$/year, i = 12%/year,
ε = 6000 units/\$, S_o = \$2/unit, L = 1 year, C_s = \$500/order,
d = \$1/unit (or 10% of regular unit cost)

Optimal Decisions and Results

Regular Decisions		Regular Results			OTOS Decisions		OTOS Results				$\Delta\Pi_L$ as % of	$\Delta\Pi_L$ as % of
P_r	Q_r	R_r	Π_{cr}	Π_{Lr}	P_s	Q_s	R_s	Π_{cs}	Π_{Ls}	$\Delta\Pi_L$	Π_{Lr}	Π_{cr}
13.43	516	19,419	1,545	52,706	13.16	2,097	21,031	6,456	53,592	886	1.68%	57.33%

As Table 1 shows, under our parameter values, in the regular situation, the retailer's optimal retail price works out to be \$13.43/unit. The corresponding demand is 19,419 units/year, and the optimal order quantity is 516 units/order (or less than two weeks' supply). These optimal decisions result in a regular cycle profit (in NPV terms) of \$1,545 and a lifetime NPV of \$52,706.

Now, assume that the manufacturer has offered an OTOS discount of \$1/unit (i.e., 10% of the regular unit cost), available at the time of the retailer's next order. Also assume that because it requires the use of a different model and involves the need to communicate a special price to the customers, the retailer's ordering cost for the special order, is \$500, instead of the regular \$100. Table 1 shows that, in this situation, the retailer's special order quantity would be 2,097 units and his special selling price would be \$13.16/unit. Given the special price, during the OTOS cycle, the retailer would experience a demand rate of 21,031 units/year. Thus, the special order quantity will last for approximately five weeks. The retailer's profit (in NPV terms) from the OTOS cycle will be \$6,456.

However, this special cycle NPV is not directly comparable with the regular cycle NPV of \$1,545 since the two cycles involve different lengths of time. The product's lifetime NPV from the special cycle followed by all regular cycles is \$53,592. Thus, the retailer's net increase in lifetime NPV due to the OTOS is \$886. In comparison to the NPV of all regular cycles (\$52,706), this net advantage looks small (1.68%). However, \$886 is 57% of a single regular cycle's NPV of \$1,545. This short-term advantage is very attractive. After all, the OTOS decisions are short term, one-cycle decisions. In short, our numerical example shows that if a retailer adopts our model, he would obtain some long-term gain and a particularly attractive short-term gain.

Of course, conclusions from a numerical example are only as valid as the assumed parameters. Hence, in what follows, we provide an analysis of the sensitivity of our results to each one of the assumed parameters. The numerical example in Table 1 serves as the base case for this sensitivity analysis.

SENSITIVITY ANALYSIS

The only parameter we hold constant throughout the sensitivity analysis is the retailer's regular unit cost of the product. However, changes in some of the other parameters could be seen as relative changes in the unit cost.

Table 2

Sensitivity to d, the OTOS Discount Per Unit

	Regular Decisions		Regular Results			OTOS Decisions		OTOS Results				$\Delta\Pi_L$ as % of Π_{Lr}	$\Delta\Pi_L$ as % of Π_{cr}
d	P_r	Q_r	R_r	Π_{cr}	Π_{Lr}	P_s	Q_s	R_s	Π_{cs}	Π_{Ls}	$\Delta\Pi_L$		
0.4	13.4	516	19,419	1,545	52,706	13.33	1,098	20,049	3,060	52,629	-77	-0.15%	-4.98%
0.8	13.4	516	19,419	1,545	52,706	13.22	1,746	20,698	5,248	53,202	496	0.94%	32.11%
1.0	13.4	516	19,419	1,545	52,706	13.16	2,097	21,031	6,456	53,592	886	1.68%	57.33%
2.0	13.4	516	19,419	1,545	52,706	12.87	4,170	22,776	13,811	56,746	4,040	7.66%	261.48%

Holding other assumed parameters at their values in Table 1, in Table 2 we vary the assumed amount of discount offered by a supplier to the reseller. As can be seen, when the discount is only $0.40 (or 5% of the normal unit cost), using special OTOS policies would result in a net loss to the reseller. Thus, the reseller is better off continuing to use his regular policies during the OTOS period and simply benefiting from the windfall gain from the discounted cost. However, as the amount of discount (and its percentage of normal unit cost) increases, the OTOS strategies become increasingly attractive, both, from the long term and the short-term perspective. When the discount is as large as 25% of the normal unit cost, the reseller may want to use a special order quantity that is 8 times his regular order quantity and pass on more than a fourth of his unit cost saving to his customers. Such a one-time opportunity can increase the reseller's lifetime NPV by 7.66% and his single cycle net advantage can be several times his normal single cycle profit.

Similarly, we carried out a detailed examination of the sensitivity of our results to each one of the parameters of our model. In Table 3, we provide a brief summary of the alternative values of parameters used, the resulting indices of long term and short-term advantage of the optimal OTOS strategies. As would be expected, the results are highly sensitive to the price elasticity. The greater the price elasticity, the greater are the advantages of optimal OTOS strategies.

Table 3

Sensitivity Analysis			
Changed Parameter	**$\Delta\Pi_L$ as % of Π_{Lr}**	**$\Delta\Pi_L$ as % of Π_{cr}**	**Comment**
ε = 5,000	1.09%	41.72%	The results are highly sensitive to this parameter. The greater the price elasticity of demand, the greater are the short term and long term advantages of optimal OTOS decisions.
ε = 6,000	1.68%	57.33%	
ε = 7,000	2.75%	80.48%	
C_s = 200	2.25%	76.74%	The results are highly sensitive to this parameter. The greater the cost of ordering an OTOS order, the smaller are the short term and long term advantages of optimal OTOS decisions.
C_s = 500	1.68%	57.33%	
C_s = 800	1.11%	37.91%	
L = 0.75	1.80%	51.46%	The results are moderately sensitive to this parameter. As the *expected life of the product* increases, long term advantages of optimal OTOS decisions decrease while short term advantages increase.
L = 1.00	1.68%	57.35%	
L = 1.50	1.47%	64.46%	
S_o = 1.00	1.55%	54.67%	Short term results are not too sensitive to this parameter. However, long term results are rather sensitive. As the *salvage value of the product* increases, both short term and long term advantages of optimal OTOS decisions increase.
S_o = 2.00	1.68%	57.35%	
S_o = 3.00	1.83%	60.24%	
C_r = 50	1.25%	60.16%	Short term results are not too sensitive to this parameter. However, long term results are rather sensitive. As the ordering cost of a regular order increases, long term advantages of optimal OTOS decisions increase, but short term advantages decline.
C_r = 100	1.68%	57.33%	
C_r = 200	2.41%	58.08%	
H = 0.5	1.82%	60.01%	The results are not too sensitive to this parameter. The greater the *fixed holding cost per unit* of inventory, the smaller are the short term and the long term advantages of OTOS optimal decisions.
H = 1.0	1.68%	57.33%	
H = 1.5	1.56%	54.85%	
h = 0.02	1.75%	58.53%	The results are not too sensitive to this parameter. The greater the *fixed holding cost per dollar* of inventory, the smaller are the short term and the long term advantages of optimal OTOS decisions.
h = 0.05	1.68%	57.33%	
h = 0.08	1.62%	56.17%	
i = 0.08	1.77%	59.95%	The results are not too sensitive to this parameter. The greater the *annual cost of capital,* the smaller are the short term and the long term advantages of optimal OTOS decisions.
i = 0.12	1.68%	57.35%	
i = 0.16	1.61%	54.96%	

The results are also highly sensitive to the ordering cost of the special order. As the ordering cost of the special order increases, the advantage of the special OTOS policies declines. From a practical point of view this is rather important to understand. In the past, researchers have assumed that, in an OTOS, there would be no change in the ordering cost. That assumption is not only unrealistic; it inflates the advantage attributable to optimal OTOS policies.

The results are moderately sensitive to the product's expected life. As expected life increases, the short-term advantages of the OTOS policies increase while the long-term advantages decline. Note also that as a product's salvage value increases the OTOS decisions are increasingly advantageous both in the long run and in the short run. Thus, it seems that OTOS decisions are more beneficial for non-obsolescent products than they are for obsolescent products.

Finally, Table 3 indicates that, from both, the long-term and the short-term perspectives, the results are not very sensitive to changes in regular ordering costs, holding costs, or cost of capital.

CONCLUSION

Today many products are characterized by price elasticity, sudden obsolescence, and short expected lives. Drawing on the relevant literature, first we built a model for a retailer's optimal price and order quantity decisions for such products in the regular situation, i.e., in the absence of a one-time-only discount. Our model is comprehensive. It allows for price elasticity, accounts for a variety of types of inventory holding costs, and deals with obsolescence costs and capital costs in precise and theoretically correct manner. Because manufacturers and vendors of S-Obs products often offer a one-time-only discount for such products, we then extended our model to accommodate the OTOS situations. Because a special order requires the use of a different model and additional costs of announcing a price change, our model used an explicitly different ordering cost for the special OTOS order. A numerical example showed that the use of our model could provide some long-term gain and a particularly attractive short-term improvement in a retailer's profit. A retailer stands to benefit the most from our model if the discount is substantial, the product demand is highly price sensitive, and the retailer's ordering cost for the special order is substantial. Also, it seems that OTOS decisions are more beneficial for non-obsolescent products than they are for obsolescent products.

There are several directions for further work on this topic. First, we assumed a linear and deterministic demand function. A non-linear function is likely to be more realistic in most situations. An extension of our model to a non-linear demand relationship should be straight forward, particularly since we do not derive any closed-form solutions but depend on Excel® to solve the problem. Also, a stochastic demand function is more likely to be encountered in real life. An extension to allow for a stochastic demand function would be relatively more complicated but doable.

A critical assumption in our model is that the fact that a manufacturer has offered an OTOS does not in itself change a retailer's assessment of the product's life expectancy and/or salvage value. In real life, a retailer may assume, often correctly, that an OTOS is a signal of an imminent obsolescence. Thus, in view of the OTOS, a retailer's perceived life expectancy and/or salvage value for the product may be reduced. On the other hand, as we have pointed out, an impending price increase with one last opportunity to buy at the lower price is a situation that is mathematically equivalent to an OTOS. In such situations of impending price increase, a retailer may deduce that the product's life expectancy may be greater than his original estimate of that expectancy. In any case, an extension of our model to allow for such a change in the perceived life expectancy of a product offered on an OTOS would also be an interesting and productive direction for further work. Finally, two recent models of OTOS situations for non-obsolescent products suggest that a retailer's optimal strategy is not to sell the entire special order quantity at the special price. Instead, a retailer should sell only a portion of his special order quantity at a special price, reverting to his regular selling price for the remaining portion of the special order quantity (Abad, 2003; Arcelus, Shah & Srinivasan, 2003). It seems that this type of a strategy may be optimal also for products subject to sudden obsolescence.

REFERENCES

Abad, P. L. (2003). Optimal price and lot size when the supplier offers a temporary price reduction over an interval. *Computers and Operations Research, 30*, 63-74.

Arcelus, F.J., N. H. Shah & G. Srinivasan. (2003). Retailer's Pricing, Credit and inventory policies for deteriorating items in response to temporary price/credit incentives. *International Journal of Production Economics,* 81-82, 153-152.

Ardalan, A. (1994). Optimal prices and order quantities when temporary price discounts result in increase in demand. *European Journal of Operational Research, 72*, 52-61.

Ardalan, A. (1995). A comparative analysis of approaches for determining optimal price and order quantity when a sale increases demand. *European Journal of Operational Research, 84*, 416-430.

Aucamp, D. C. & P. J. Kuzdrall. (1986). Lot sizes for one-time-only sales. *Journal of Operational Research Society, 37*, 79-86.

Barbosa, L. C. & M. Friedman. (1979). Inventory lot size models with vanishing markets. *Journal of the Operations Research Society, 30*(2), 1129-1132.

Brown, G. W., J. Y Lu & R. J. Wolfson. (1964). Dynamic modeling of inventories subject to sudden obsolescence. *Management Science, 11*(1), 51-63.

Brown, R. G. (1967). *Decision rules for inventory management.* New York: Holt, Rinehart & Winston.

Brown, R. G. (1982). *Advanced Service Parts Management.* Norwitch, VT: Materials Management Systems, Inc.

Hadley, G. & T. M. Whitin. (1963). *Analysis of Inventory Systems.* Englewood Cliffs, NJ: Prentice-Hall.

Hill, A. V., V. Girard & V. A. Mabert. (1989). A decision support system for determining optimal retention stocks for service parts inventories. *IIE Transactions, 21*(3), 236:248.

Joglekar, P. & P. Lee. (1993). An exact formulation of inventory costs and optimal lot size in face of sudden obsolescence. *Operations Research Letters, 14*, 283-290.

Joglekar P. & P. Lee. (1996). A profit maximization model for a retailer's stocking decisions on products subject to sudden obsolescence, *Production and Operations Management, 5*(3), 288-294.

Lev, B. & H. J. Weiss. (1990). Inventory models with cost changes. *Operations Research, 38*, 53-63.

Masters, J. M. (1991). A note on the effect of sudden obsolescence on the optimal lot size. *Decision Sciences, 22*, 1180-1186.

Naddor, E. (1966). *Inventory Systems*, New York: John Wiley & Sons.

Nahmias, S. (1974). Inventory depletion management when the field life is random. *Management Science, 20*, 1276-1284.

Nose, T., H. Ishii & T.Nishida. (1984). Perishable inventory management with stochastic lead-time and different selling prices. *European Journal of Operations Research, 18*, 322-338.

Silver, E. A. & R. Peterson. (1985). *Decision Systems for inventory management and production planning*. New York: John Wiley & Sons.

Taylor, S. G. & C. E. Bradley. (1985). Optimal ordering strategies for announced price increases. *Operations Research, 33*(2), 312-325.

Tersine, R. J. & E. T. Grasso. (1978). Forward buying in response to announced price increases. *Journal of Purchasing and Materials Management, 14*(2), 20-22.