

A Comprehensive Set of Models of Intra and Inter-organizational
Coordination for Marketing and Inventory Decisions in a Supply Chain

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Abstract

This paper presents a set of eight models of coordination for pricing and order quantity decisions in a supply chain consisting of one manufacturer and one retailer of a product with price sensitive demand. In many organizations, the marketing department makes the pricing decisions while the operations department makes the order quantity decisions. Yet, many researchers have suggested that organizations can benefit from intra-organizational coordination for these two decisions. Similarly, in a typical supply chain, the manufacturer's decisions are not coordinated with the retailer's decisions. Yet, many researchers have suggested that a supply chain can benefit from coordination of the order quantity decisions of the manufacturer and the retailer. Others have recommended supply chain coordination for pricing decisions. Thus, there are a number of possibilities for intra and inter-organizational coordination (or a lack of coordination) for the pricing and order quantity decisions in a supply chain. We study each possibility and compare its advantages and disadvantages relative to other coordination possibilities. The analysis leads to interesting and, at times, paradoxical results. For example, we find that, in the absence of inter-organizational coordination, intra-organizational coordination by either the manufacturer, or the retailer, or both, leads to a reduction in the supply chain's profit compared to its profit from a no coordination situation. As would be expected, complete intra and inter-organizational coordination results in the best profit for the supply chain. However, the supply chain's profit from inter-organizational coordination for pricing decisions alone is only marginally smaller than the profit from complete coordination. Hence, considering the tangible and intangible costs of a coordination mechanism, we recommend that a supply chain should coordinate its pricing decisions but should not indulge in the coordination of its order quantity decisions. An extensive sensitivity analysis confirms our major findings and yields interesting insights into the relative advantages and disadvantages of various coordination possibilities in marketing and inventory related decisions.

Keywords: Supply Chain, Intra and Inter-organizational Coordination, Channel Coordination, Departmental Coordination, Pricing Decisions, Order Quantity Decisions

1. Introduction

Supply chain management is the process of planning, executing and controlling the interdependencies of activities carried out by different supply chain members or business units in order to create value for the end customer (Lambert et al., 1998). Researchers in several business disciplines have recommended intra and inter-organizational coordination for effective management of a supply chain. Economists have long argued that in a decentralized system, local objectives may be in conflict with total system objectives and locally optimized decisions may be suboptimal for the overall system. Porter (1985) and other strategy researchers have emphasized that when activities and decisions in a value chain (within and between firms) are inconsistent and uncoordinated, the value chain loses its competitive advantage. Some information systems scholars (Curran and Ladd, 2000; Sandoe et al., 2001) have recommended the use of an Enterprise Resource Planning (ERP) software to avoid information inconsistency at various levels of decision-making within a firm. Other information systems researchers have observed the benefits of a collaborative process in which technologically sophisticated retailers pool the local level expertise and information with the manufacturers (Messinger and Narasimhan, 1995; Adams, 1995). Forecasting scholars note the “bullwhip” effect in which the fluctuations in order quantities for a product increase as they move up an uncoordinated supply chain from the retailer to the wholesaler to the manufacturer (Chopra and Meindl, 2001). The bullwhip effect results in increased inventories, poorer product availability and reduced profits at each level of a supply chain. Indeed, the bullwhip effect has also been linked to the boom and bust cycles in several industries. In this paper, we focus on intra and inter firm coordination for price and order quantity decisions in a supply chain.

Almost fifty years ago, combining the concepts from price theory with those from the Economic Order Quantity (EOQ) model, Whitin (1955) showed that for a product with price-elastic demand, a retailer who coordinates the price and order quantity decisions would obtain a greater profit than one that allows two separate departments (marketing and operations) to make these two decisions in an uncoordinated manner. Independently, Kunreuther and Richard (1971) found the same results for inter-departmental coordination at a retailer as well as a manufacturer. Eliashberg and Steinberg (1993) provide a good summary of this idea. Over the years, Whitin’s (1955) and Kunreuther and Richard’s (1971) works have served as foundations to a stream of research (Tersine and Price, 1981; Arcelus and Srinivasan, 1987; Ardalan, 1991; Hall, 1992;

Martin, 1994; Arcelus and Srinivasan, 1998; and Abad, 2003) that confirms the same findings in a variety of circumstances.

Recommendations for inter-organizational coordination for price and order quantity decisions come from two different perspectives: the marketing perspective and the operations perspective. The marketing perspective originated in Jeuland and Shugan's (1983) work. They showed that inter-organizational coordination for marketing decisions (price, promotion, etc.) led to a lower price for the consumer and higher volume and profit for a supply chain. Although, they recognized that, in real life, there were many roadblocks to attaining this coordination, Jeuland and Shugan (1983) suggested that a quantity discount (more correctly, "annual volume discounts" rather than "discounts for quantity per order") scheme was one possible mechanism for channel coordination. Over the years, numerous researchers (McGuire and Staelin, 1983; Moorthy, 1987; Jeuland and Shugan, 1988; Ingene and Parry, 1995) have confirmed that Jeuland and Shugan's (1983) findings about the advantages of channel coordination are correct under their assumption of no product substitutability. However, McGuire and Staelin (1983) found that for highly competitive products (characterized by product substitutability) a decentralized system performed better than a vertically integrated system. Jeuland and Shugan (1988) showed that, under certain circumstances, rational conjectures could help profit maximizing channel members to attain some degree of coordination without a formal contractual arrangement. Moorthy (1987) suggested a simpler coordination method by using a two-part tariff and showed that a quantity discount schedule was not necessary for channel coordination. A wide variety of pricing schemes, including quantity surcharges can do the job. Ingene and Parry (1995) extended Jeuland and Shugan's (1983) work to the case of multiple retailers. By and large, the marketing perspective ignored inventory related costs of a product in modeling the profits of a supply chain.

In contrast, the operations perspectives focused on the inventory related costs of a product while assuming that the product's demand was not price-sensitive. Although Crowther (1964) had shown that a wholesaler (who could be a manufacturer as well) could increase his profits by managing the timing of the retailer's orders using quantity discounts, the works of Monahan (1984) and Lal and Staelin (1984) represent the earliest works that looked at channel coordination from the operations perspectives. Monahan (1984) used an accounting method, valid for vertically integrated firms, to propose an *all-units* quantity discount scheme while Lal and Staelin (1984) used an accounting method, valid for non-vertically integrated firms, to propose an *incremental* quantity discount scheme. Both of these approaches showed that a

manufacturer (or wholesaler) using a properly designed quantity discount scheme (i.e., a price reduction for ordering a quantity that is larger than the retailer's individually optimal quantity per order) could increase his profits without hurting the retailer's profits. In other words, a properly designed quantity discount scheme could increase a supply chain's total profit. A number of works (Rosenblatt and Lee, 1985; Banerjee, 1986a and 1986b; Lee and Rosenblatt, 1986; Goyal, 1987; Dada and Srikanth, 1987; Kohli and Park, 1987; Joglekar, 1988; Joglekar and Tharthare, 1990) have further refined and extended these models. Joglekar (1988) questioned the practical usefulness of Monahan's (1984) and Lal and Staelin's (1984) approaches of focusing on inventory costs alone and ignoring any marketing considerations such as price elasticity. He pointed out that these approaches could not explain commonly observed quantity discounts such as 10% of a product's unit costs, instead they explained discounts of the order of $3/10^{\text{th}}$ of 1% that were perhaps too small to justify the vendor's costs of announcing the discounts and the buyer's (or retailer's) costs of analyzing the desirability of the discounts. Noting that Monahan (1984), Banerjee (1986a) and others had warned of the difficulties of actually implementing a coordination scheme, Joglekar and Tharthare (1990) proposed an "individually responsible and rational" approach to increasing a channel's profits through purely voluntary actions on the part of a vendor and a retailer.

More recently, some researchers have attempted to combine the marketing and the operations perspectives. Weng and Wong (1993) and Weng (1995a, 1995b) integrated Jeuland and Shugan's (1983) concepts of price-elastic demand with Monahan's (1984) and Lal and Staelin's (1984) concerns for inventory related costs and confirmed that coordinated decision-making benefited both the supplier and the buyer and maximized the joint profit. They showed that Monahan's (1984) optimal all units quantity discount policy was equivalent to Lal and Staelin's (1984) optimal incremental quantity discount policy. However, they also showed that a quantity discount scheme alone was not sufficient to guarantee joint profit maximization. An additional annual franchise fee paid by the buyer to the vendor was necessary to ensure the maximization of supply chain profits. Boyaci and Gallego (2002) also provided a comprehensive and integrated view of the literature on the marketing and the operations perspectives on wholesaler-retailer coordination for price and order quantity decisions. They showed that earlier conclusions about the desirability of wholesaler-led or jointly negotiated quantity discount policies depended crucially on the implicit assumption of retailer owned inventory. They found that an optimal coordinated policy could be implemented as a nested replenishment policy where

the retailer pays the wholesaler the unit wholesale price only when the retailer sells the items. By adjusting the wholesale price, this inventory consignment policy could distribute the gains of channel coordination without requiring any side payments or franchise fee. Furthermore, Boyaci and Gallego (2002) showed that the practice of separating the pricing decisions from the inventory decisions was near optimal for systems with high demand rates. While most of the earlier literature had focused on a one-wholesaler-one-retailer situation, Boyaci and Gallego (2002) confirmed their results in the one-wholesaler-many-retailers case as well.

Although available research has considered various aspects of departmental coordination and channel coordination, no one has done a comprehensive assessment of all possible scenarios of coordination within and across a supply chain's members. In this paper, we break down the coordination problem to its lowest common denominator by considering every sensible scenario for intra and inter-organization coordination for price and order quantity decisions. We consider eight different scenarios. In model I, we assume no coordination whatsoever. Models II, III and IV assume no inter-organizational coordination. However, they consider intra-organizational coordination at the manufacturer, at the retailer, and at both the manufacturer and retailer, respectively. Models V, VI and VII assume no intra-organizational coordination but inter-organizational coordination for the order quantity decisions, for the pricing decisions, and for both the order quantity and the pricing decisions, respectively. Finally, in model VIII we assume total intra and inter organizational coordination. Figure 1 presents a schematic summary of the eight models considered in this study.

Insert Figure 1 Here

Through this comprehensive approach, we obtain a fuller perspective on the relative advantages of different forms of coordination and some very interesting insights. Ironically, we find that in the absence of inter-organizational coordination, intra-organizational coordination at either or both members of the channel results in lower total system profits than those under no coordination. Inter-organizational coordination for order quantity decisions (but not for pricing decisions) results in a practically insignificant (0.09% in our numerical example) increase in the supply chain profit compared to the profit from the no coordination situation. A supply chain's highest profit comes from total intra and inter-organizational coordination and shows a sizable (19.55% in our numerical example) increase in the supply chain's profits compared to the profits under no coordination. However, inter-organizational coordination for pricing decisions only results in a supply chain profit that is only marginally (0.07%) smaller than the profit under total

coordination. Hence, considering that coordination for any purpose is itself difficult and costly, we recommend that supply chain partners should strive to coordinate their pricing decisions only. Our sensitivity analysis confirms these findings under alternative values of the model parameters.

This paper is organized as follows. Section 1 has already presented a review of the relevant literature. Section 2 presents our mathematical notation. In Section 3, we present each one of our models along with the solution procedures used to solve them. Section 4 presents a base case numerical example for all the scenarios and discusses a number of important findings based on that numerical example. Section 5 presents a sensitivity analysis that generates additional insights into the desirability of alternative forms of supply chain coordination. The final section summarizes our conclusions and outlines some directions for future research.

2. The Channel Coordination Problem and Notation

The basic objective of this paper is to compare the relative merits of various forms of intra and inter-organizational coordination for price and order quantity decisions. We assume that all decision-makers are rational and seek to optimize their respective objectives. We further assume that if two or more decision-makers agree to coordinate their decisions, they agree on a redefined objective that represents their joint interests. From that point on, they act as a single decision-maker seeking to optimize that objective. We consider a supply chain consisting of a single retailer and a single manufacturer (or wholesaler) of a product with price sensitive demand. This simple bilateral monopoly structure allows us to focus on those merits and demerits without the distraction of multiple products, multiple retailers, multiple manufacturers, uncertainty, asymmetric power struggles or institutional constraints. In this first attempt at developing a comprehensive set of models of supply chain coordination, we also keep our models simple by assuming a stable and linear demand function. The retailer faces holding costs for the product's inventory, ordering costs for placing orders with the manufacturer, selling and other variable costs per unit of the product sold, and an annual fixed cost for the facilities and organization to carry the product. The retailer must determine the optimal order quantity and the optimal price to charge the consumer.

The manufacturer faces holding costs, ordering costs, production setup costs, per unit production costs, and an annual fixed cost to have the machinery and organization to produce the product. The demand for the manufacturer is the same as that for the retailer. The

manufacturer's problem is to determine an optimal production lot size (given the manufacturer's production capacity per year – which can be infinite when the “manufacturer” is only a wholesaler) and an optimal price to charge to the retailer. Although the literature offers several different approaches to the determination of the manufacturer's lot size in such a situation, as discussed later, we have chosen to use Joglekar's (1988) approach of assuming that the manufacturer's lot size will be an integer multiple of the retailer's order quantity. Hence, the manufacturer's problem is to determine that optimal multiple for his production lot size and the optimal price to charge the retailer. Assume the following relevant parameters for the retailer:

H_r = Holding costs per unit of inventory per year

Note: We assume that the retailer's holding cost is independent of the transfer price between the manufacturer and retailer. In our paper, this transfer price can vary from model to model, and we do not want the retailer's actual holding costs to be distorted by the changes in the transfer price.

O_r = Ordering cost per order

V_r = Retailer's variable (selling and other) costs per unit

F_r = Retailer's annual fixed costs for the facilities and organization to carry this product (including such costs as advertising, display, and supervision.)

P_r = Retail price charged to the consumer ($P_r = V_m + G_m + V_r + G_r$)

e = Coefficient of the product's demand elasticity

D = Annual demand for the product at price P_r , ($D = A - eP_r$)

A = A constant, representing demand at $P_r=0$

The retailer's decision variables are:

G_r = Retailer's variable profit margin ($G_r = P_r - P_m - V_r$)

Q_r = Retailer's order quantity per order

Also, let us define the manufacturer's relevant parameters as:

H_m = Holding costs per unit of inventory per year regardless of the value of the inventory

Note: Rather than assuming that the manufacturer's holding cost is proportional to the cost of the item, we assume it to be constant per unit simply to keep the manufacturer's model consistent with the retailer's model.

O_m = Order Processing cost for the manufacturer per order of the retailer

- V_m = Variable production cost per unit (e.g., raw materials, labor, etc)
- F_m = Fixed costs of the manufacturer for the facilities and organization for the production of this product (e.g., plant depreciation, supervision, etc.)
- P_m = Price charged by the manufacturer to the retailer ($P_m = V_m + G_m$)
- S_m = Manufacturer's setup cost per production lot for this product
- Q_m = Manufacturer's production lot size ($Q_m = KQ_r$)
- R_m = Manufacturer's annual production capacity for this product (R_m assumes an infinite value if the "manufacturer" is simply a wholesaler)

The manufacturer's decision variables are G_m and K :

- G_m = Variable profit margin ($G_m = P_m - V_m$)
- K = The integer multiple used to determine the production lot size given the retailer's order quantity

Furthermore, we define the relevant annual costs and profits of the manufacturer and retailer as follows:

- Π_r = Marketing profit of the retailer ($\Pi_r = G_r D - F_r$)
- Π_m = Marketing profit of the manufacturer ($\Pi_m = G_m D - F_m$)
- Π = Total marketing profit of the supply chain ($\Pi = \Pi_r + \Pi_m$)
- C_r = Inventory related costs of the retailer
- C_m = Inventory related cost of the manufacturer
- C = Total inventory related cost of the supply chain ($C = C_m + C_r$)
- Z_r = Net profit of the retailer ($Z_r = \Pi_r - C_r$)
- Z_m = Net profit of the manufacturer ($Z_m = \Pi_m - C_m$)
- Z = Total net profit of the supply chain ($Z = Z_m + Z_r$)

The goal of this research is to investigate a comprehensive set of eight different models based upon varying assumptions concerning the existence or lack of channel coordination as well as the existence or lack of coordination between the production and marketing departments of each channel member.

In many organizations, pricing decisions are made by the marketing departments while order quantity decisions are made by the operations (i.e., either the production or the procurement) departments. As a result of this uncoordinated decision making in the supply chain,

the marketing departments of the retailer and the manufacturer respectively try to maximize their “marketing profits,” Π_r and Π_m , given by the following equations:

$$\Pi_r = G_r D - F_r \quad (1)$$

$$\Pi_m = G_m D - F_m \quad (2)$$

In response to the demand resulting from these pricing decisions, the retailer’s operations department decides on the order quantity by focusing on minimizing its inventory related cost consisting of inventory holding and ordering costs. These inventory-related costs are given by:

$$C_r = \frac{O_r D}{Q_r} + \frac{H_r Q_r}{2} \quad (3)$$

Next, in response to the retailer’s order quantity, the manufacturer’s operations department determines its production lot size. The literature on inventory policy coordination is rich with numerous models for this situation. Monahan (1984) assumed that a manufacturer’s production lot size is identical to the retailer’s order quantity. Joglekar (1988) pointed out that a better strategy is to have the manufacturer’s lot size as an integer multiple of the retailer’s order quantity. Goyal (1995) argued that a coordinated system can do even better if the manufacturer produces a large lot and ships varying sized sub-batches to the retailer. Viswanathan (1998) showed that neither Joglekar’s (1988) equal size sub-batch policy nor Goyal’s (1995) unequal sub-batch policy was dominant under all circumstances. Hill (1997) presented a more general class of manufacturer’s optimal policy for the coordinated situation. While Goyal’s (1995) and Hill’s (1997) models work for a coordinated system, they do not have equivalent models for the uncoordinated situation. On the other hand, Joglekar’s (1988) model can work for the coordinated and the uncoordinated situation. Since our focus is on a comparison of the coordinated and the uncoordinated systems, here we shall use Joglekar’s (1988) approach of assuming that the manufacturer’s lot size as an integer multiple of the retailer’s order quantity.

The manufacturer’s objective is to minimize the sum of annual production setup costs, order processing costs and inventory carrying costs. These inventory-related costs are given by:

$$C_m = \frac{S_m D}{K Q_r} + \frac{O_m D}{Q_r} + \frac{H_m Q_r}{2} \left[(K-1) - \frac{(K-2)D}{R_m} \right] \quad (4)$$

When there is intra-organizational coordination only (as suggested by Whitin, 1955 or Kunreuther and Richard, 1971), the retailer’s goal is to maximize Z_r while the manufacturer’s goal is to maximize Z_m . When there is intra-organizational coordination for price decisions only

as suggested by Jeuland and Shugan's (1983) marketing approach, the marketing departments of the manufacturer and retailer seek to maximize the joint marketing profit, Π , while their operations departments seek to minimize the respective inventory related costs, C_m and C_r . When there is intra-organizational coordination for order quantity decisions only, the operations departments of the manufacturer and retailer seek to minimize the joint inventory costs, C , while their marketing departments seek to maximize the respective marketing profits, Π_m and Π_r . Finally, when there is total coordination in the supply chain (as suggested by Boyaci and Gallego, 2002), the goal would be maximize Z .

3. Channel Coordination Models

Model-I: No Coordination

In the first model, we assume that there is no channel coordination and no departmental coordination, either. Thus, each channel member (the manufacturer and the retailer) acts independently of the other in selecting its profit margin and order quantity. Also, the operations and the marketing departments of each channel member act independent of one another and choose optimal decisions on the basis of their departmental criteria.

The condition for the maximization for the retailer's marketing profit, Π_r of Equation (1), is obtained by setting $\frac{\partial \Pi_r}{\partial G_r} = 0$. This results in an optimal G_r given by:

$$G_r = \frac{A}{2e} - \frac{V_m + G_m + V_r}{2} \quad (5)$$

Given this optimal G_r , the retailer's operations department minimizes its inventory related costs,

C_r of Equation (3), by setting $\frac{\partial C_r}{\partial Q_r} = 0$.

The resulting optimal Q_r is equal to:

$$Q_r = \sqrt{\frac{2DO_r}{H_r}} \quad (6)$$

Similarly, the manufacturer's marketing profit, Π_m of Equation (2), is maximized when

$$G_m = \frac{A}{2e} - \frac{V_m + V_r + G_r}{2} \quad (7)$$

And the manufacturer's inventory related costs, C_m of Equation (4), are minimized when

$$K = \frac{1}{Q_r} \sqrt{\frac{2DS_m}{H_m \left(1 - \frac{D}{R_m}\right)}} \quad (8)$$

We should remind the reader that K must be a positive integer. Hence, when Equation (8) yields a non-integer value, one would have to evaluate the two integers above and below that value to choose the one that gives the smallest C_m .

Now, even though in Model I we are assuming no coordination within or between the supply chain members, if there is to be a stable solution to the pricing and order quantity decisions, we must assume that each decision-maker finds the parameters assumed (and consequently, the optimal decisions made) by the other decision-makers to be consistent with his/her own assumptions. In the absence of such system-wide consistency, as each decision-maker goes through several iterations of his decisions in response to the assumed parameter values revealed by the decisions made by the other parties, there may not be a stable state situation at all and the iterations could continue in an endless loop.

Note that although we are suggesting that a decision-maker reacts to the assumed parameter values revealed by others' decisions, unlike Jeuland and Shugan (1988), we are not assuming that a decision-maker has any conjectures about how other parties' decisions may or may not change in response to his/her own decisions. Note also that we are not requiring that the assumed system parameters must be the true parameters, just that the parameter values assumed by the various decision-makers must be mutually consistent. Thus, the true elasticity of demand may be 4,000 units per dollar reduction in retail price when the demand elasticity assumed is only 3,600 units per dollar reduction in retail price. However, as long as all the decision makers assume the same value (3,600) for the demand elasticity, and each decision-maker is rational, Model I can yield a stable solution for the system.

Thus, even in a no coordination situation, we require mutually consistent parameter values assumed by various decision-makers. When we move on to our subsequent models involving coordination within or across channel members, the need for the consistency of assumed parameters will be greater since the coordinating decision-makers must first agree on explicitly stated values of the relevant parameters. Hence, in all of our models, we presume that all the assumed system parameters (the demand function and the variety of costs incurred by the manufacturer and the retailer) are mutually consistent.

Now, assuming system-wide consistency of assumed parameters and decisions, in Model I, solving Equations (5) to (8) simultaneously results in Equations (9) to (12) below:

$$G_r^* = G_m^* = \frac{A}{3e} - \frac{V_m + V_r}{3} \quad (9)$$

$$Q_r^* = \sqrt{\frac{2D^*O_r}{H_r}} \quad (10)$$

$$K^* = \frac{1}{Q_r^*} \sqrt{\frac{2D^*S_m}{H_m \left(1 - \frac{D^*}{R_m}\right)}} \quad (11)$$

Where

$$D^* = A - e(V_m + G_m^* + V_r + G_r^*) \quad (12)$$

Thus, Equations (9) to (12) provide a closed form solution to the stable state optimal decisions for the no coordination situation. Once the assumed parameters of a situation are known, G_r^* , G_m^* , and Q_r^* can be obtained by Equations (9) and (10). Equation (11) gives an initial value of K^* . However, note that K^* must be a positive integer. Hence, when Equation (11) yields a non-integer value, one should evaluate the two integers above and below that value to choose the one that gives the smallest C_m . Alternatively, one can use Excel Solver to solve these equations subject to an integer value constraint on K^* . In this paper, we have used this latter approach.

Notice from Equation (9) that the optimal profit margin of each channel member is equal to that of the other. Furthermore, the optimal profit margin of each member depends upon the parameters of the demand function as well as the unit variable costs of both members. Thus, whether the relevant information is actively shared or not, in the equilibrium state, each channel member must know the demand function and the assumed variable cost of each member. Similarly, Equations (10) and (11) indicate that, in the equilibrium state, each departmental decision maker must know the relevant optimal decisions of the other parties.

Note that our solution is an equilibrium solution in the full game-theoretic meaning of the term. At our equilibrium, no decision-maker would be able to improve upon his/her objective by changing his/her decisions while other decision-makers stick to their optimized decisions. This is true not just with Model I but with all of our models as long as one recognizes that when two or more decision-makers agree to coordinate, they act as a single decision-maker.

Model-II: Intra-organizational Coordination at the Manufacturer Only

This model assumes that the manufacturer and the retailer act independently of each other to select their profit margins and order/production quantities. As in Model I, its uncoordinated marketing and operations departments make the retailer's decisions. However, unlike Model I, the pricing and production lot size decisions of the manufacturer are coordinated as recommended by Kunreuther and Richard's (1971) model.

The coordinated manufacturer seeks to maximize his net profit, $Z_m (= \Pi_m - C_m)$, given by:

$$Z_m = G_m D - F_m - \left[\frac{S_m D}{K Q_r} + \frac{O_m D}{Q_r} + \frac{H_m Q_r}{2} \left((K-1) - \frac{D(K-2)}{R_m} \right) \right] \quad (13)$$

Optimizing Z_m with respect to G_m and K , results in the following equations:

$$\frac{\partial Z_m}{\partial G_m} = -e G_m + A - e(V_m + G_m + V_r + G_r) + \frac{e S_m}{K Q_r} + \frac{e O_m}{Q_r} - \frac{e H_m Q_r (K-2)}{2 R_m} = 0 \quad (14)$$

Or

$$G_m = \frac{A}{2e} - \frac{V_m + V_r + G_r}{2} + \frac{S_m}{2K Q_r} + \frac{O_m}{2Q_r} - \frac{H_m Q_r (K-2)}{4R_m} \quad (15)$$

And

$$\frac{\partial Z_m}{\partial K} = \frac{S_m D}{K^2 Q_r} - \frac{H_m Q_r}{2} \left[1 - \frac{D}{R_m} \right] \quad (16)$$

Or

$$K = \frac{1}{Q_r} \sqrt{\frac{2DS_m}{H_m \left(1 - \frac{D}{R_m} \right)}} \quad (17)$$

As in Model I, the uncoordinated retailer's decisions would continue to be governed by Equations (5) and (6).

Unfortunately, in Model II, even after assuming system-wide consistency of parameters and decisions, one cannot arrive at a simpler set of equations for determining the stable state solutions. Thus, the stable state solution to this model must be arrived at through an iterative computer procedure. We begin with a set of initial values of G_r , G_m , Q_r , and K . Then, using Equations (5) and (6) we find the corresponding optimal value of G_r , and Q_r . These values are fed back in Equations (15) and (17) to find the corresponding optimal value of G_m , and K . The process is repeated until there are no changes in the resulting values (rounded to two decimals) of

G_r , G_m , Q_r , and K from iteration to iteration. At this point, Equation (17) gives an initial value of K^* . However, since K^* must be a positive integer, when Equation (17) yields a non-integer value, one has to evaluate the two integers above and below that value to choose the one that gives the largest Z_m . For our numerical examples, we have used Excel to implement this and other computational procedures.

Model-III: Intra-organizational Coordination at the Retailer Only

This model assumes that the manufacturer and the retailer act independently of each other to select their profit margins and order/production quantities. As in Model I, the uncoordinated marketing and operations departments make the manufacturer's decisions. However, unlike Model I, the pricing and order quantity decisions of the retailer are coordinated as recommended by Whitin's (1955) model.

The net profit of the retailer is given by:

$$Z_r = G_r D - F_r - \left(\frac{O_r D}{Q_r} + \frac{H_r Q_r}{2} \right) \quad (18)$$

Optimizing Z_r with respect to G_r and Q_r , results in the following equations:

$$\frac{\partial Z_r}{\partial G_r} = -eG_r + A - e(V_m + G_m + V_r + G_r) + \frac{eO_r}{Q_r} = 0 \quad (19)$$

Or

$$G_r = \frac{A}{2e} - \frac{V_m + G_m + V_r}{2} + \frac{O_r}{2Q_r} \quad (20)$$

And

$$\frac{\partial Z_r}{\partial Q_r} = \frac{eO_r}{Q_r^2} - \frac{H_r}{2} = 0 \quad (21)$$

Or

$$Q_r = \sqrt{\frac{2DO_r}{H_r}} \quad (22)$$

As in Model I, here also, Equations (7) and (8) provide the conditions for the optimal values of G_m , and K .

As in Model II, in Model III, even after assuming system-wide consistency of parameters and decisions, one cannot arrive at a simpler set of equations for determining the stable state solution. Thus, the stable state solution to this model must be arrived at through an iterative

computer procedure. We begin with a set of initial values of G_r , G_m , Q_r , and K . Then, using Equations (20) and (22) we find the corresponding optimal value of G_r , and Q_r . These values are fed back in Equations (7) and (8) to find the corresponding optimal value of G_m , and K . We use Excel Solver to ensure that only integer values of K are considered. The process is repeated until there are no changes in the resulting values (rounded to two decimals) of G_r , G_m , Q_r , and K from iteration to iteration.

Model-IV: Intra-organizational Coordination at the Manufacturer and the Retailer

In this model we continue with the assumption that the manufacturer and the retailer act independently of each other. However, we assume that the pricing and order quantity decisions at each channel member is coordinated, so that the net profit of the manufacturer, $Z_m = \Pi_m - C_m$, and that of the retailer, $Z_r = \Pi_r - C_r$, are maximized independently. Now, the conditions for the optimal decisions of the manufacturer are given by Equations (15) and (17) while those for the retailer are given by Equations (20) and (22).

As in the case of Models II and III, the stable state solution to this model is found through an iterative computer procedure. Beginning with a set of initial values of G_r , G_m , Q_r , and K , the corresponding optimal value of G_r and Q_r can be found using Equations (20) and (22). These values are fed back in Equations (15) and (17) to find the corresponding optimal value of G_m , and K . Excel Solver enforces the required integer value for K . The process is repeated until there are no changes in the resulting values (rounded to two decimals) of G_r , G_m , Q_r , and K from iteration to iteration.

In the following models, we consider the cases of inter-organizational coordination.

Model-V: Inter-organizational Coordination for Order Quantity Decisions Only

In Model V, we assume that the operations departments of the manufacturer and the retailer coordinate their order quantity decisions with each other to minimize the system's total inventory related costs (as suggested by the operations approach discussed earlier). However, in setting their respective profit margins, the marketing departments of the manufacturer and the retailer act independently of each other and independently of their operations departments.

As in Model I, here Equation (9) gives the optimal values of G_r^* and G_m^* . In this case, the focus is on minimizing the system's total inventory related costs. Hence, separate values of C_r , C_m , are no longer relevant. The manufacturer and retailer would have to find some other mutually agreeable way of sharing the total inventory related costs similar to various quantity

discount schemes suggested by Dolan (1978), Lal and Staelin (1984), Dada and Srikanth (1987), and Monahan (1984) for coordinating the inventory related decisions of the manufacturer and the retailer. Total inventory related costs are given by:

$$C = \frac{S_m D}{K Q_r} + \frac{O_m D}{Q_r} + \frac{H_m Q_r}{2} \left[(K-1) - \frac{D(K-2)}{R_m} \right] + \frac{O_r D}{Q_r} + \frac{H_r Q_r}{2} \quad (23)$$

Setting $\frac{\partial C}{\partial Q_r} = 0$ and $\frac{\partial C}{\partial K} = 0$, results in the following relationships:

$$Q_r = \sqrt{\frac{2D^* \left(O_m + O_r + \frac{S_m}{K} \right)}{\left[H_r + H_m \left[(K-1) - (K-2) \left(\frac{D^*}{R_m} \right) \right] \right]}} \quad (24)$$

$$K^* = \frac{1}{Q_r^*} \sqrt{\frac{2D^* S_m}{H_m \left(1 - \frac{D^*}{R_m} \right)}} \quad (25)$$

Where $D^* = A - e(V_m + G_m^* + V_r + G_r^*)$.

Thus, in Model V, Equations (9), (24) and (25) provide the closed form equilibrium solution to the situation. Of course, given the requirement that K must be an integer, it is best not to use Equations (24) and (25) to find the Q_r and K values. Instead, given the optimal values of G_r^* and G_m^* from Equation (9), we use Excel to minimize the C of Equation (23) by varying Q_r and K values, subject to the requirement that K must be a positive integer.

Model-VI: Inter-organizational Coordination for Pricing Decisions Only

In Model VI, we assume that the marketing departments of the manufacturer and the retailer coordinate their profit margin decisions to maximize their joint marketing profit (as suggested by the marketing approach discussed earlier) but the operations departments of the manufacturer and the retailer minimize their respective inventory related costs independently of each other and independently of their marketing departments. In this case, since the focus is on the joint marketing profit, it is assumed that the break down of that joint profit between the two channel members is not important. In other words, the price charged by the manufacturer to the retailer would not be the mechanism by which the joint marketing profit would be shared by the two channel members. Instead, the manufacturer and the retailer would find some other mutually agreeable method of sharing their joint marketing profit such as the quantity discount schemes or

a two-tier pricing system proposed by Jeuland and Shugan (1983), Moorthy (1987), Ingene and Parry (1995), and others. Hence, in this case, we shall simply assume that the manufacturer sells the product to the retailer at his variable cost without adding any profit margin. That is, here $G_m = 0$.

In this case, the focus is on maximizing the system's total marketing profit. Hence, separate values of Π_r and Π_m are no longer relevant. The joint marketing profit of the manufacturer and the retailer is given by:

$$\Pi = G_r D - F_m - F_r \quad (26)$$

Setting $\frac{\partial \Pi}{\partial G_r} = 0$, we obtain the optimal value of G_r as:

$$G_r^* = \frac{A}{2e} - \frac{V_m + V_r}{2} \quad (27)$$

Now, the retailer's inventory related costs are given by:

$$C_r = \frac{O_r D}{Q_r} + \frac{H_r Q_r}{2} \quad (28)$$

Hence, the retailer's operations department's optimal order quantity will be obtained by setting

$\frac{\partial C_r}{\partial Q_r} = 0$. This results in:

$$Q_r^* = \sqrt{\frac{2D^* O_r}{H_r}} \quad (29)$$

The manufacturer's inventory related costs are still given by Equation (4). Hence, the manufacturer's operations department finds that Equation (8) provides the formula for the optimal value of K , given the fact that K must be a positive integer. Thus, in this model, Equations (27) and (29) give the optimal values of G_r^* and Q_r^* . Then we search for the integer value of K that minimizes the manufacturer's inventory related costs of Equation (4). As always we use Excel solver to obtain the optimal solutions.

Model-VII: Inter-organizational Coordination for Order Quantity and Pricing Decisions

In Model VII, as in Model VI, we assume that in setting their prices, the marketing departments of each of the two channel members coordinate with each other, but not with their operations departments. However, unlike Model VI, here we assume that, given those prices, the /operations departments of the manufacturer and the retailer coordinate with each other to determine their

order quantities. Thus, this model assumes that the marketing approach and the operations approach to inter-organizational coordination are both implemented but independently of each other.

As in Model VI, here we assume that $G_m = 0$, and the optimal G_r^* is given by Equation (27). Also, as in Model VI, separate values of Π_r and Π_m are not relevant. Finally, as in Model V, separate values of C_r and C_m are not relevant. The system's total inventory related costs are given by Equation (23), and we use Excel to minimize the C of Equation (23) by varying Q_r and K values, subject to the requirement that K must be a positive integer.

Model-VIII: Total Intra and Inter-organizational Coordination

In model VIII, we assume that the two channel members fully integrate their decision-making within as well as across the channel. Their joint objective is to maximize the total net profit of the system given by:

$$Z = G_r D - \frac{O_m D}{Q_r} - \frac{S_m D}{K Q_r} - \frac{H_m Q_r}{2} \left[(K-1) - \frac{D(K-2)}{R_m} \right] - \frac{O_r D}{Q_r} - \frac{H_r Q_r}{2} \quad (30)$$

As in Models VI and VII, an implied assumption here is that $G_m = 0$. Also, here separate values of C_r , C_m , Π_r , Π_m , Z_r and Z_m are not relevant. The manufacturer and retailer must find alternative mutually agreeable method of sharing the total profit.

Setting $\frac{\partial Z}{\partial G_r} = 0$, $\frac{\partial Z}{\partial Q_r} = 0$, and $\frac{\partial Z}{\partial K} = 0$, results in the following equations:

$$G_r = \frac{A}{2e} - \frac{V_m + V_r}{2} + \frac{1}{2Q_r} \left(O_m + O_r + \frac{S_m}{K} \right) - \frac{H_m Q_r}{4R_m} (K-2) \quad (31)$$

$$Q_r = \sqrt{\frac{2D \left(O_m + O_r + \frac{S_m}{K} \right)}{\left[H_r + H_m \left[(K-1) - (K-2) \left(\frac{D}{R_m} \right) \right] \right]}} \quad (32)$$

$$K = \frac{1}{Q_r} \sqrt{\frac{2DS_m}{H_m \left(1 - \frac{D}{R_m} \right)}} \quad (33)$$

These equations can be solved simultaneously. However, such a procedure may yield a fractional value for K . Hence, we simply let Excel Solver maximize Equation (30) by varying G_r , Q_r and K values, subject to the requirement that K must be a positive integer.

4. Numerical Example

In the foregoing section, we have presented a comprehensive set of eight models of coordination within and across the members of a supply chain. Considering a manufacturer and a retailer dealing with a price sensitive product, we developed equations to assess relevant consequences of each channel member's selling price and order quantity decisions from each member's point of view as well as from the supply chain's point of view. We noted that a condition for the determination of stable state decisions and consequences of each model is that the system parameters assumed by the various decision-makers must be mutually consistent. Given that condition, we outlined the procedures for finding the optimal decisions and consequences of each one of our models. Now, we turn to a numerical example to assess the relative merits and demerits of each type of supply chain coordination.

Table 1 summarizes our base case numerical example. In the top section of Table 1, we outline the assumptions common to all eight models. Demand as a function of retailer's price is assumed to be given by the formula $D = 50000 - 4000P_r$. The retailer's assumed costs parameters are a fixed cost of \$10,000/year, a variable cost of \$1.25/unit, ordering cost of \$250/order, and holding costs of \$1.00/unit/year. The manufacturer's production capacity is assumed to be 30,000 units/year. The manufacturer's assumed cost parameters are a fixed cost of \$20,000/year, a variable cost of \$1.50/unit, an order processing cost of \$100/retailer's order, a set up cost of \$600/production set up, and holding costs of \$0.50/unit/year.

Insert Table 1 Here

The next section of Table 1, labeled Part 1, provides the stable state optimal decisions and consequences of the retailer and the manufacturer in the type of coordination visualized by each model. Finally, in order to understand the relative differences among our eight models, in Part 2 of Table 1, we present the decisions and consequences of each model as a percentage of the corresponding decisions and consequences in Model I.

Looking at the optimal decisions first, we see that, in the no-coordination situation (Model I), the optimal profit margin for each channel member is \$3.25/unit. In Model I, the retailer's optimal order quantity is 2,550 units/order and the manufacturer's production lot size is 3 times that order quantity or 7,650 units/set up. As can be noted, the optimal profit margins and order quantities in Models II, III, and IV (when there is only internal coordination at one or both channel members but no coordination across the channel) show only minor variations from the optimal decisions of Model I (the no-coordination situation), both in absolute and in percent

terms. Note that in Models II, III, and IV we have marked some profit and cost entries as NA since under the coordinated approaches of these models such entries are meaningless. For example, since model II assumes maximization of Z_m through coordinated profit margin and order quantity decisions at the manufacturer, separate values of Π_m and C_m are meaningless. If a manufacturer were interested in assessing individual department's performance separately, he would have to find some other means to do so. Similarly, in Model III, separate values of Π_r and C_r are meaningless. In Model IV, Π_m , C_m , Π_r and C_r are all meaningless.

In Model V, where there is no internal coordination but there is coordination across the channel only to minimize system wide inventory related costs, as would be expected, the optimal profit margins of the retailer and the manufacturer are identical to those in Model I. However, retailer's optimal order quantity and manufacturer's optimal production lot size are substantially different from those in Model I. In this model, C_m and C_r are meaningless.

In Models VI and VII, where there is no internal coordination but there is coordination across the channel to maximize the marketing profit, the manufacturer's profit margin is assumed to be 0 and the retailer's profit margin reflects the total profit margin of the supply chain. It should be noted that, in these cases, the supply chain's total profit margin (given by G_r^*) is significantly smaller than the total margin in Models I to V (given by $G_m^* + G_r^*$). Consequently, in Models VI to VIII, P_r , the price paid by the consumer is substantially smaller (approximately 82.5% of Model I price) which results in substantial increase in the product's demand (approximately 150% of Model I demand). In turn, this results in a substantial increase in the retailer's optimal order quantity (Q_r^*) and a corresponding increase in manufacturer's lot size (KQ_r^*). The difference between Model VI and Model VII is that Model VI assumes no coordination across the channel to minimize system wide inventory related costs while Model VII assumes such coordination. Thus, while the profit margins in the two models are identical, compared to Model VI, the retailer's order quantity and the manufacturer's lot size show some increase in Model VII. In Model VI, Π_m , Π_r , Z_m , and Z_r are all meaningless. In Model VII, Π_m , Π_r , C_m , C_r , Z_m , and Z_r are all meaningless.

Model VIII visualizes a situation of total coordination on the part of the retailer and the manufacturer to maximize the supply chain profit net of its inventory related costs. As in Models VI and VII, here the manufacturer's profit margin is assumed to be 0 and the retailer's profit margin is the supply chain's total profit margin. As can be seen from Table 1, now the supply chain's profit margin is slightly greater than that in Models VI and VII. Consequently, the retail

price paid by the consumer is slightly greater which results in some decrease in the product's demand. In turn, this results in some decrease in the retailer's optimal order quantity and the manufacturer's lot size compared to Model VII. In Model VIII, Π_m , Π_r , C_m , C_r , C , Z_m , and Z_r are all meaningless.

Of course, the most interesting comparison of the models lies in the consequences of their respective stable state decisions. In Model II, with internal coordination at the manufacturer only, as would be expected, the manufacturer's net profit (Z_m) is greater than what it is Model I (but only by 1.13%). However, the retailer's net profit (Z_r) is reduced by an amount greater than the manufacturer's gain (in percent terms, the reduction is 1.35%). Thus, the supply chain's total net profit (Z) is smaller (by 0.36%) in Model II compared to Model I. With internal coordination at the retailer only, in Model III the results are very similar to those in Model II, except that in Model III, as would be expected, the retailer's net profit (Z_r) shows a slight increase which is accompanied by a slightly greater reduction in the manufacturer's net profit (Z_m).

In model IV we visualize internal coordination at each channel member but no coordination across the channel. As can be seen, in this situation, contrary to what one would expect, the supply chain's total net profit (Z) is smaller than what it is in either Model II or model III and smaller yet compared to the profit in Model I. It is also interesting to note that when both channel members coordinate their internal decision-making, each member makes a smaller net profit compared to the situation of no coordination. Thus, our models have discovered an important paradox of supply chain coordination. Agrawal and Tsay (2001) discovered the same paradox in the context of coordination in a standard newsvendor situation involving order quantity decisions in face of stochastic demand during a fixed inventory cycle time.

Models II to IV together illustrate another paradox of supply chain coordination. One would expect that when a supply chain's total net profit (Z) decreases, the consumer must have benefited from a lower retail price. However, as we can see, the stable state retail prices (P_r) in each of these models are slightly higher than those in Model I. Thus, when there is only internal coordination, either at a single member or at both, but no across the channel coordination, the consumer as well as the supply chain as a whole are worse off compared to a no coordination situation!

Note further that our models have not formally assumed any cost for the very organization and implementation of the necessary coordination procedures implied by our various models. However, as several authors (including Jeuland and Shugan, 1983; Monahan,

1984; Banerjee, 1986a, Joglekar, 1988; Joglekar and Tharthare, 1990) have noted, coordination among two or more decision-makers is neither easy nor costless. Coordinated decision-making requires information sharing and associated costs and it foregoes some of the advantages of functional specialization.

In uncoordinated situations, operations and marketing managers perform specialized functions with associated economies of scale. An operations manager may be concerned about many inventory items that must be ordered (some from the same supplier), stocked in a limited warehouse space, using a limited budget. A marketing manager must decide on the selling price of each item but also on other marketing variables such as advertising, shelf space allocations for each item while simultaneously considering the tie-ins among the various items' demands and constraints on the total shelf space and advertising budget. Focusing on a single product's price and order quantity decisions, a centralized decision-maker must ignore such functional considerations across several products. Also, a centralized decision-maker may not have the know-how to manage both the operations and the marketing functions as efficiently as the functional managers could. Even when information on relevant parameters is fully shared (as assumed by our stable state conditions), coordinated decision-making requires changing the locus of decision-making, development of a new decision model, and communication and enforcement of the coordinated decision (Joglekar, 1988). Considering such costs of coordination, it should be clearer yet that internal coordination on the part of members of a supply chain (in absence of any across the channel coordination) is harmful to the consumer as well as to the supply chain.

Model V visualizes a situation of across the channel coordination for minimizing inventory related costs. As noted before, a number of researchers have recommended such coordination in the past (Dolan, 1978; Lal and Staelin, 1984; Dada and Srikanth, 1987; Monahan, 1984; Banerjee, 1986b; Lee and Rosenblatt, 1986; Goyal, 1987). Although these researchers assumed a price insensitive demand, Model V seems to be consistent with their research since the optimal selling prices and the resultant product demand in Model V are identical to those in Model I. As would be anticipated, in Model V, the supply chain as a whole does attain a saving in the total inventory related costs (C). However, that saving is rather modest (0.84% in our numerical example) and the percent growth in the supply chain's net profit (Z) is practically insignificant (0.09%). As we have said before, coordination is neither easy nor costless. Integrating the key business processes across the supply chain is a more difficult task

than coordinating them within a firm. The two firms' many constituencies' objectives may have little in common resulting in potential conflict and inefficiencies for the supply chain (Sherman, 1998). Conflicting objectives preclude managers from effectively managing trade-offs across functions as well as across companies (Lee and Billington, 1992). Thus, in an empirical study of nine industries, D'Aveni and Ravenscraft (1994) found that the economies of vertical integration are somewhat offset by increased bureaucracy costs. Hence, given the marginal improvement in the supply chain's net profit shown by Model V, we believe that, in general, coordination across the channel for the minimization of inventory related costs only is not worth pursuing.

Of course, we must admit that our inventory related costs include only the costs of ordering and carrying the inventory of a product with a stable and predictable demand curve. Supply chain scholars justify the need for across the channel coordination of inventory related decisions based on several other phenomena such as the bullwhip effect in the case of products with less predictable demand (Lee and Whang, 1999; Chopra and Meindl, 2001). Some of those justifications may be valid in situations that are different from the one we have considered here.

Models VI, VII, and VIII all involve across the channel coordination in determining the final retail price for the product. As can be seen from Table 1, all of these models result in a significant decrease (approximately 17.5% in our numerical example) in the price that the consumer has to pay (P_r). Thus, from the consumer's perspective, across the channel marketing coordination is highly desirable. A commonly held belief is that what is good for the consumer is not likely to be good for the supply chain. Paradoxically, across the channel marketing coordination is highly desirable from the supply chain's perspective as well. As Table 1 shows, the supply chain's total net profit (Z) in each one of Models VI, VII, and VIII is significantly larger (approximately 19.5% in our numerical example) than the net profit in Model I. Thus, even after allowing for the fact that the economies of vertical integration will be somewhat offset by increased bureaucracy costs (D'Aveni and Ravenscraft, 1994), the gains in profit estimated by our Models VI, VII, and VIII are sizable.

As is theoretically expected, Model VIII, which assumes fully centralized decision making, produces the best profit for the supply chain. Model VII, which assumes across the channel coordination in pricing, accompanied by across the channel coordination in order quantity decisions, produces the second best total profit for the supply chain (Z). However, note that the supply chain profit produced by Model VI, which requires across the channel coordination in pricing decisions only, is only $7/100^{\text{th}}$ of 1% smaller than the profit given by

Model VIII. As we have argued before, we believe that the difficulties and costs of coordination in the implementation of Models VII or VIII would be much greater than those in the implementation of Model VI. Hence, from the supply chains perspective, Model VI may be most desirable. This finding is consistent with Boyaci and Gallego's (2002) finding that the practice of making marketing decisions that ignore operations costs, followed by operations decisions that take retail price as given is fully justified.

Finally, since the retail price in Model VI is the same as the retail price in Model VII but lower than that in Model VIII, from the consumer's perspective also, Model VI seems to be the most desirable one of all. Thus, one of the paradoxes our models have discovered is that what is good for the supply chain is also good for the consumer!

5. Sensitivity Analysis

In order to ensure that our conclusions are not based purely on the chosen numerical values of the parameters in Table 1 (referred to as the base case from here on), we conducted a sensitivity analysis by increasing the numerical values of each one of our base case parameters, one parameter at a time, by 10%. In each one of these analyses, we found that a particular change resulted in approximately the same percentage change in the supply chain's total net profit (Z) in each one of our models. Thus, we believe that our Section 4 conclusions about the relative merits of different forms of supply chain coordination are valid over a wider range of values of the parameters of our models.

The sensitivity analysis also showed that our numerical results are most sensitive to the chosen values of A and e . For example a 10% increase in e leads to approximately 22 to 24% decrease in the supply chain's total profit in each one of our models, while a 10% increase in A leads to approximately 22 to 24% increase in the supply chain's profit. These equal but opposite impacts make sense since, from the optimality conditions of the various models, we know that optimal values of G_r and G_m are influenced by the ratio of A to e . Thus, our models' sensitivity to A can be easily deduced from an analysis of sensitivity to e . We believe it is important to take a good look at our models' sensitivity to e since that parameter is perhaps the most difficult one to accurately estimate. Hence, in the next sub-section, we present a detailed analysis of the sensitivity of our models to e .

The supply chain's profits in our models are also somewhat sensitive to changes in V_m and V_r . A 10% increase in V_m or V_r leads to a 4 to 5% reduction in the profits of various models.

However, V_m and V_r are perhaps the parameters that are easiest to estimate accurately. The impacts of 10% increases in the rest of the parameters of our models (namely, O_m , S_m , R_m , H_m , O_r , and H_r) are practically insignificant (less than 1%) on the supply chain's total profit in all of our models. As such, we do not formally present an analysis of the sensitivity of our models to 10% increases in parameters other than e .

5.1 Sensitivity to e

Table 2 presents the optimal decisions and results of a situation when e is 110% of the base case e . In order to facilitate a clear comparison, Table 2 consists of two parts. Part 1 provides the numerical values of the optimal decisions and consequences when all the parameter values are identical to those in the base case, except e , which is now set as $e = 4,400$ units/dollar. In Part 2 of Table 2, comparing the numbers in Part 1 with the corresponding numbers in Table 1, for each one of the decisions and consequences of our models, we present the percent changes caused by the 10% increase in e .

Insert Table 2 Here

As can be seen from the comparison of Tables 1 and 2, as e increases by 10%, in Model I, the optimal profit margins of the manufacturer (G_m^*) and the retailer (G_r^*) each drop from \$3.25/unit to \$2.87/unit (a decline of 11.66%). Models II to V display approximately the same percent declines in G_r^* and G_m^* . Recall that in Models VI to VIII, G_m^* is assumed to be 0. However, in those models also, G_r^* shows an approximately the same percent decline.

In each one of the models, the optimal value of Q_r^* declines by 1.5 to 1.8%. In all the models, K^* values are unaffected by the increase in e .

P_r , the price paid by the consumer, declines by approximately 8.15% in Models I to V but by approximately 7.4% in Models VI to VIII, where to begin with, the optimal prices were substantially lower in the base case. In all of our models, the 10% increase in e and the corresponding decrease in optimal P_r , together, lead only to a 2.8 to 3.2% decrease in the product's annual demand. Thus, it is interesting to note that overall, the optimized product demand is not very sensitive to changes in e .

In model I, the 10% increase in e results in a 26.86% decline in the manufacturer's marketing profit, Π_m , and an 18.53% decline in the retailer's marketing profit, Π_r . While separate marketing profits for each channel member are not relevant in several of our models, where relevant, Π_m , and Π_r display percentage declines that are similar to those in Model I. Thus,

marketing profits of both the members are highly sensitive to e , and the manufacturer's marketing profit is even more sensitive.

In contrast, the manufacturer's inventory related costs, C_m , decline by only approximately 1.12% and the retailer's inventory related costs, C_r , decline by 1.42% in Model I. In most of our models, either C_m or C_r , or both, are not relevant. However, where they are relevant, they show similar very small percent declines. Thus, the retailer's optimized inventory related costs are not very sensitive and manufacturer's optimized inventory related costs are even less sensitive to changes in e .

The sensitivity of the net total profits of the manufacturer and the retailer (Z_m and Z_r) is naturally dictated by the sensitivity of Π_m , Π_r , C_m , and C_r . As can be seen from Table 2, in Models I to IV, a 10% increase in e results in 30 to 32% reduction in Z_m and approximately 20% reduction in Z_r . Thus, in those models, both the manufacturer's and the retailer's net profits are highly sensitive to e , the manufacturer's profit being relatively more sensitive. The break down between Z_m and Z_r is not relevant for Models V to VIII. However, the supply chain's total profit, $Z = Z_m + Z_r$, is most relevant in all of our models. As can be seen from Table 2, a 10% increase in e leads to a 22.6 to 24.5% decline in the supply chain's total profit in all of our models. Thus, the supply chain's total profit is highly sensitive to e .

Note also that since that the effects of a 10% increase in e on the supply chain's total profit is approximately the same in percentage terms across all of our models, our Section 4 conclusions about the relative merits of the various models are fully valid in this situation as well. As we have asserted before, our sensitivity analysis on the other parameters of our model also found that our base case conclusions about the relative merits of various forms of coordination remain valid in each one of those parameter changes. In short, our sensitivity analysis confirms that:

- (a) Contrary to what one would expect, when both channel members coordinate their internal decision-making, each member makes a smaller net profit compared to the situation of no coordination.
- (b) In the absence of across the channel coordination, internal coordination on the part of members of a supply chain is harmful to the consumer as well as the supply chain.
- (c) Coordination across the channel for the minimization of inventory related costs only might not be worth pursuing, particularly when one considers the organizational costs of coordination.

(d) Across the channel marketing coordination (for determining the price) is highly desirable from the supply chain's perspective as well as the consumer's perspective. In this sense, what is good for the supply chain is also good for the consumer.

In addition, our analysis has shown that in all of our models, a supply chain's total profits are highly sensitive to e , the demand elasticity. Recognizing that e is perhaps the parameter in our model that is most difficult to accurately estimate, we now turn to an analysis of the effects of an inaccurate estimate of e in the various situations of coordination we have modeled.

5.2 An assessment of the consequences of an inaccurate estimate of e

In Part 1 of Table 3, we present the "optimal" decisions (of all the decision makers, coordinated or otherwise) and the consequences to a supply chain that incorrectly assumes $e = 4,000$ units/dollar when the true $e = 4,400$ units/dollar. Note that the decisions G_r^* , G_m^* , Q_r^* , and K^* , used in Table 3 are identical to those in Table 1. However, the consequences in Table 3 are based on $e = 4,400$ units/dollar. Then, comparing the values in Table 3 with the optimal decisions and consequences of a situation where e is known to be 4,400 units/dollar (presented earlier in Part 1 of Table 2), in Part 2 of Table 3, we present for each model, the percent differences caused by the supply chain's incorrect estimation of the e value.

Insert Table 3 Here

As can be seen from Part 2 of Table 3, there are significant differences in the incorrect decisions (based on an incorrect assumption of the e value) and the optimal decisions (based on the correct value of e). Correspondingly, there are also significant differences in the various consequences. While we could discuss the results in Table 3, one row at a time, by now our reader knows how to read those numbers and draw appropriate conclusions. Here, we only want to emphasize that in Models I to V, the incorrect estimate of e results in over 30% reduction in the supply chain's total net profit (Z). On the other hand, in Models VI to VIII, that incorrect estimate of e leads to a reduction of only 2.43 to 3.11% in Z . In other words, the costs of an incorrect estimate of e are enormous in supply chains that use Models I to V (involving no across the channel coordination for marketing decisions), but considerably smaller in supply chains that use Models VI to VIII (involving across the channel coordination for marketing decisions). Thus, across the channel coordination seems to be the best defense against likely inaccuracies in the estimate of the demand elasticity.

In this context, it is important to note the recent work by Hendricks and Singhal (2003) who estimated the shareholder wealth effects of (mostly uncoordinated) supply chain glitches that resulted in production or shipment delays (often caused by incorrect estimates of demands). Based on a sample of 519 glitches announced during 1989-2000, Hendricks and Singhal (2003) found that these glitches were associated with an abnormal decrease in shareholder value of 10.28%. Although this estimate is not as high as our estimate of a 30% reduction in an uncoordinated supply chain's profit due to a 10% mis-estimation of demand elasticity, it certainly gives credence to our model since:

- (1) The glitches that Hendricks and Singhal (2003) studied may not be due to a 10% mis-estimation in demand.
- (2) Some of the glitches in the sample could be related to factors other than mistakes in demand estimation.
- (3) Although correlated, a company's profits do not directly reflect in the shareholder value.

Finally, we want to note that, in percent terms, the impact of an inaccurate estimate of e on the Z value is smaller in Model VI than it is in Model VIII. Thus, once again, across the channel coordination for marketing decisions only (i.e., without the coordination for the order quantity decisions) seems to be the best strategy for a supply chain.

6. Conclusion and Future Research Directions

We developed and analyzed a comprehensive set of eight different models based upon varying assumptions concerning the existence or lack of intra and inter-organizational coordination between the operations and marketing departments of each member of a supply chain. We also conducted a comprehensive sensitivity analysis by increasing each one of our model parameters by 10% while keeping the other parameters constant. Our models turned out to be highly sensitive to the assumed value of price elasticity, somewhat sensitive to the unit variable costs of the manufacturer and the retailer and not at all sensitive to the rest of the parameters. The sensitivity analysis showed that a particular change in any parameter increased or decreased the supply chain's total profit by approximately the same percentage in all of our models. Thus, the conclusions about the relative merits of various forms of coordination that we drew using our base case analysis seem to be valid under a wider range of values of parameters.

Because our models are highly sensitive to the assumed price elasticity of demand, we presented a detailed analysis of that sensitivity. We also examined the likely costs of a misestimate of the value of demand elasticity.

Based on all of our analysis, we conclude that

1. When only one channel member coordinates its internal decisions, that member makes a greater profit compared to a no coordination situation, however, the resultant reduction in the profit of the other member is greater yet. Thus, internal coordination at any one member of the channel actually reduces the total channel profit.
2. Contrary to what one would expect, when both channel members coordinate their internal decision-making, each member, and the supply chain as a whole, makes a smaller net profit compared to the situation of no coordination. This is particularly true when one considers the organizational costs of coordination that are not quantified by our models.
3. In the absence of inter-organizational coordination, internal coordination on the part of any of the members of a supply chain is also harmful to the consumer.
4. Complete inter-organizational coordination produces the highest profit for the supply chain. However, inter-organizational coordination for only price determination produces almost as great a profit for the supply chain. Hence, inter-organizational coordination to minimize inventory related costs might not be worth pursuing, particularly when one considers the difficulties and organizational costs of coordination.
5. Inter-organizational coordination for price determination only is also most desirable from the consumer's perspective, since it produces the lowest retail price. In this sense, what is good for the supply chain is also good for the consumer.
6. The costs of incorrect estimates of demand elasticity are also the lower when there is inter-organizational coordination for price determination only rather than when there is coordination for order quantity decisions as well. Thus, given that estimates of demand elasticity are often wrong in the practice, we highly recommend that a supply chain should use the coordination approach and policies of our Model VI.

Of course, these conclusions are based on a simple one-manufacturer, one-retailer channel dealing in a single product with a stable linear demand function. Our models assume that price and order quantity are the only two possible decisions that affect the profits of individual channel members as well as the total channel. They also assume mutually consistent parameter estimates on the part of all the decision makers. Our conclusions will be truly valid only when they are

verified in more realistic situations of complex supply chains with several competing manufacturers dealing in a multitude of substitutable and non-substitutable products through a number of competing retailers who have numerous decision variables (e.g., price, promotion, shelf space, order quantity, just-in-time delivery contracts, inventory financing instruments, and so on). Such a validation would require many years of work by numerous researchers. However, we believe that this paper has made a good start towards a comprehensive evaluation of the relative merits of alternative ways of coordinating a supply chain's decisions.

References

- Abad, P.L. 2003. Optimal price and lot size when the supplier offers a temporary price reduction over an interval. *Computers and Operations Research*, **30**(1), 63-74.
- Adams, D. 1995. Category management – A marketing concept for changing times, in: J. Heilbrunn ed. *Marketing Encyclopedia: Issues and trends shaping the future*.
- Agrawal, N., A. A. Tsay. 2001. Intrafirm incentives and supply chain performance. J.S. Song, D.D. Yao, eds. *Supply Chain Structures: Coordination, Information, and Optimization*. Norwell: Kluwer Academic Publishers.
- Arcelus, F.J., G. Srinivasan. 1987. Inventory policies under various optimizing criteria and variable markup rate. *Management Science*, **33**(6), 756-762.
- Arcelus, F.J., G. Srinivasan. 1998. Ordering policies under one time only discount and price sensitive demand. *IIE Transactions*, **30**(11), 1057-64.
- Ardalan, A. 1991. Combined optimal price and optimal inventory replenishment policies when a sale results in increase in demand. *Computers and Operations Research*, **18**(9), 721-730.
- Banerjee, A. 1986a. A joint economic lot size model for purchaser and vendor. *Decision Science*, **17**(3), 292-311.
- Banerjee, A. 1986b. A quantity discount pricing model to increase vendor profits. *Management Science*, **32**(11), 1513-1517.
- Boyaci, T., G. Gallego. 2002. Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. *International Journal of Production Economics*, **77**(2), 95-111.
- Chopra, S., P. Meindl. 2001. Supply chain management: strategy, planning and operations. Upper Saddle River: Prentice Hall.
- Crowther, J.F. 1964. Rationale for quantity discounts. *Harvard Business Review*, **42**(March/April), 121-127.
- Curran, T.A., A. Ladd. 2000. SAP/ R3 business blueprint: understanding enterprise supply chain management. Upper Saddle River: Prentice Hall.
- D'Aveni, R.A., D.J. Ravenscraft. 1994. Economies of integration versus bureaucracy costs: Does vertical integration improve performance? *Academy of Management Journal*, **37**(5), 1167-1206.
- Dada, M., K.N. Srikanth. 1987. Pricing policies for quantity discounts. *Management Science*, **33**(10), 1247-1252

- Dolan, R.J. 1978. A normative model of industrial buyer response to quantity discounts, in: S.C. Jain ed. *Research Frontiers in Marketing: Dialogues and Directions*. Chicago: American Marketing Association.
- Eliashberg, J., R. Steinberg. 1993. Marketing-production joint decision-making. J. Eliashberg, G.L. Lilien, eds. *Handbooks in Operations Research and Management Science*. Amsterdam: North Holland.
- Goyal, S.K. 1987. Comment on: 'A generalized quantity discount pricing model to increase supplier's profits'. *Management Science*, **33**(12), 1635-1636.
- Goyal, S. K. 1995. A one-vendor multi-buyer integrated inventory model: A comment. *European Journal of Operational Research*, **82**(1), 209-210.
- Hall, R. 1992. Price changes and order quantities: impacts of discount rate and storage costs, *IIE Transactions*, **24**(2), 104-110.
- Hendricks, K.B., V.R. Singhal. 2003. The effects of supply chain glitches on shareholder wealth. *Journal of Operations Management*, **21**(5), 501-522.
- Hill, R. M. 1997. The single-vendor single-buyer integrated production-inventory model with a generalized policy. *European Journal of Operational Research*, **97**(3), 493-499.
- Ingene, C.A., M.E. Parry. 1995. Coordination and manufacturer profit maximization: the multiple retailer channel. *Journal of Retailing*, **71**(2), 129-151.
- Jeuland, A.P., S.M. Shugan. 1983. Managing channel profits. *Marketing Science*, **2**(3), 202-210.
- Jeuland, A.P., S.M. Shugan. 1988. Channel of distribution profits when channel members form conjectures. *Marketing Science*, **7**(2), 239-272.
- Joglekar, P.N. 1988. Comments on 'A quantity discount pricing model to increase vendor profits'. *Management Science*, **34**(11), 1391-1398.
- Joglekar, P.N., S. Tharthare. 1990. The individually responsible and rational approach to economic lot sizes for one vendor and many purchasers. *Decision Sciences*, **21**(3), 492 - 506.
- Kunreuther, H., J.F. Richard. 1971. Optimal pricing and inventory decisions for nonseasonal items. *Econometrica*, **39**(1), 173-175.
- Kohli, R., H. Park. 1989. A cooperative game theory model for quantity discounts. *Management Science*, **35**(6), 693-707.
- Lal, R., R. Staelin. 1984. An approach for developing an optimal discount pricing policy. *Management Science*, **30**(12), 1524-1539.

- Lambert, D.M., J.R. Dtock, L.M. Ellram. 1998. Fundamentals of logistics management. Boston: McGraw Hill.
- Lee, H.L., C. Billington. 1992. Managing supply chain inventory: pitfalls and opportunities. *Sloan Management Review*, **33**(3), 65-73.
- Lee, H.L., M.J. Rosenblatt. 1986. A generalized quantity discount pricing model to increase supplier's profits. *Management Science*, **32**(9), 1177-1185.
- Lee, H.L., S. Whang. 1999. Decentralized multi-echelon supply chains: incentives and information. *Management Science*, **45**(5), 633-640.
- Martin, G.E. 1994. Note on an EOQ model with a temporary sale price. *International Journal of Production Economics*, **37**(2-3), 241-243.
- McGuire, T.W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Science*, **2**(2), 161-192.
- Messinger, P., C. Narasimhan. 1995. Has power shifted in grocery channel? *Marketing Science*, **14**(2), 189-223.
- Moorthy, K.S. 1987. Managing channel profits: comments. *Marketing Science*, **6**(4), 375-379.
- Monahan, J. 1984. A quantity discount pricing model to increase vendor profits. *Management Science*, **30**(6), 720-726.
- Porter, M. 1985. Competitive advantage: creating and sustaining superior performance. New York: The Free Press.
- Rosenblatt, M.J., H.L. Lee. 1985. Improving profitability with quantity discounts under fixed demand. *IIE Transactions*, **17**(4), 388-395.
- Sandoe, K., G. Corbitt, R. Boykin. 2001. Enterprise integration. New York: John Wiley and Sons.
- Sherman, R.J. 1998. Supply chain management for the millennium. Oak Brook: Warehousing Education and Research Council.
- Tersine, R.J., R.L. Price. 1981. Temporary price discounts and EOQ. *Journal of Purchasing and Materials Management*, **17**(4), 23-27.
- Viswanathan, S. 1998. Optimal strategy for the integrated vendor-buyer inventory model. *European Journal of Operational Research*, **105**(1), 38-42
- Weng, Z.K. 1995a. Channel coordination and quantity discounts. *Management Science*, **41**(9), 1509-1522.

- Weng, Z.K. 1995b. Modeling quantity discounts under General price-sensitive demand functions: optimal policies and relationships. *European Journal of Operational Research*, **86**(2), 300-314.
- Weng, Z.K., R.T. Wong. 1993. General models for the supplier's all-unit quantity discount policy. *Naval Research Logistics*, **40**(7), 971-991.
- Whitin, T.M. 1955. Inventory control and price theory. *Management Science*, **2**(1), 61-68.

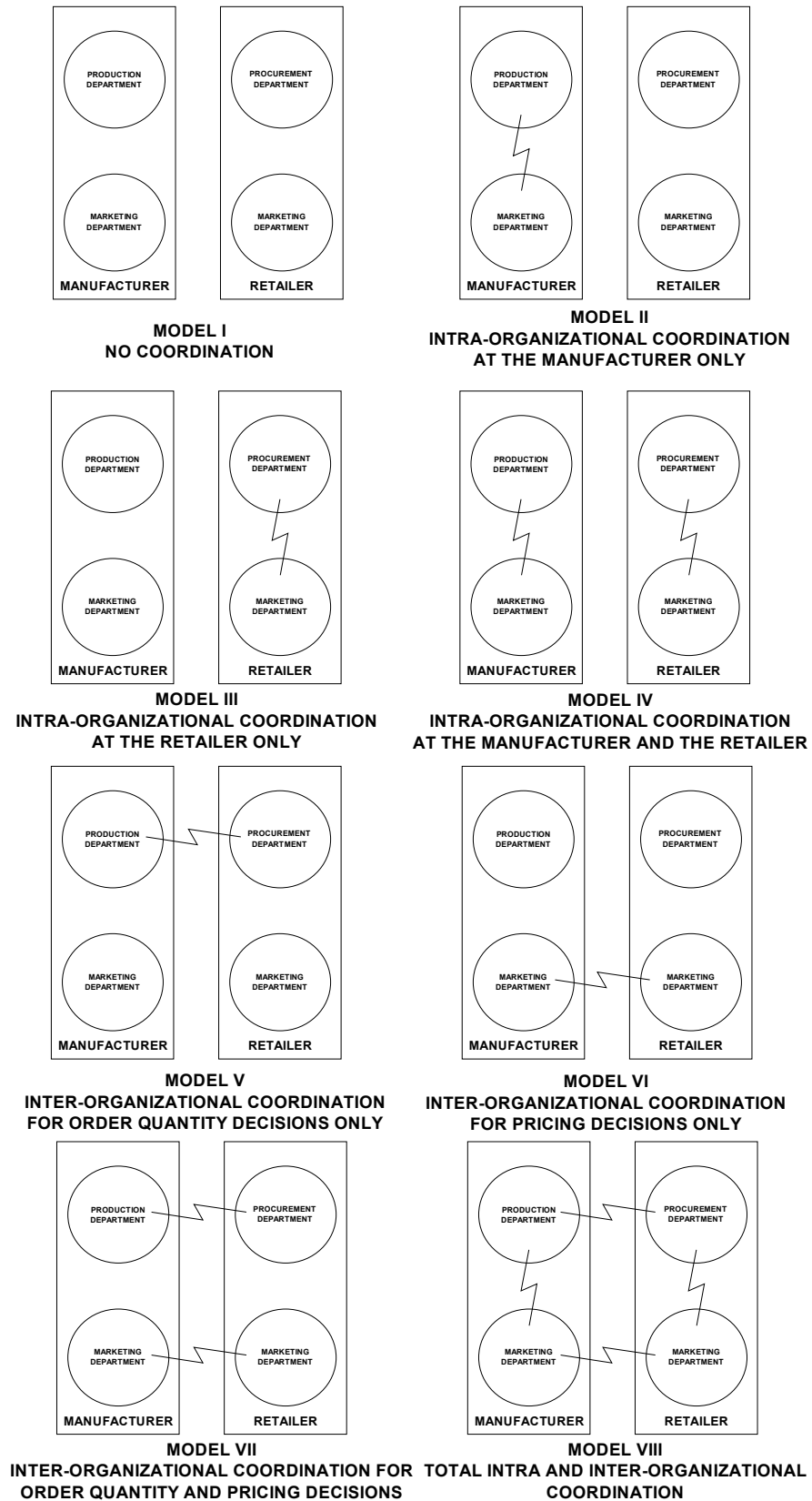


Figure 1: Supply chain models

Table 1: Optimized decisions and results for various models using base case assumptions

Assumptions common to all models									
$A = 50,000$ units, $e = 4,000$ units/dollar, $F_r = \$10,000$ /year, $V_r = \$1.25$ /unit, $O_r = \$250$ /order,									
$H_r = \$1.00$ /unit/year, $R_m = 30,000$ units/year, $F_m = \$20,000$ per year,									
$V_m = \$1.50$ /unit, $O_m = \$100$ /order, $S_m = \$600$ /set up, $H_m = \$0.50$ /unit/year.									
Part 1: Optimized decisions and consequences									
	Variable	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
Optimal Decisions	G_r^*	\$3.25	\$3.23	\$3.31	\$3.28	\$3.25	\$4.88	\$4.88	\$4.94
	G_m^*	\$3.25	\$3.28	\$3.22	\$3.28	\$3.25	\$0.00	\$0.00	\$0.00
	Q_r^*	2,550	2,544	2,538	2,524	3,357	3,122	3,579	3,551
	K^*	3	3	3	3	2	4	3	3
Optimal Consequences	P_r	\$9.25	\$9.26	\$9.28	\$9.32	\$9.25	\$7.63	\$7.63	\$7.69
	P_m	\$4.75	\$4.78	\$4.72	\$4.78	\$4.75	\$1.50	\$1.50	\$1.50
	D	13,000	12,942	12,886	12,740	13,000	19,500	19,500	19,247
	Π_m	\$22,250.00	NA	\$21,477.13	NA	\$22,250.00	NA	NA	NA
	Π_r	\$32,250.00	\$31,842.51	NA	NA	\$32,250.00	NA	NA	NA
	Π	\$54,500.00	\$54,311.73	\$54,126.98	\$53,637.86	\$54,500.00	\$65,062.50	\$65,062.50	NA
	C_m	\$2,528.26	NA	\$2,519.58	NA	NA	\$2,888.31	NA	NA
	C_r	\$2,549.51	\$2,543.85	NA	NA	NA	\$3,122.50	NA	NA
	C	\$5,077.77	\$5,067.72	\$5,057.91	\$5,032.10	\$5,034.88	\$6,010.81	\$5,994.06	NA
	Z_m	\$19,721.74	\$19,945.35	\$18,957.55	\$19,314.29	NA	NA	NA	NA
	Z_r	\$29,700.49	\$29,298.66	\$30,111.51	\$29,291.48	NA	NA	NA	NA
	Z	\$49,422.23	\$49,244.01	\$49,069.07	\$48,605.76	\$49,465.12	\$59,051.69	\$59,068.44	\$59,083.96
Part 2: Each model's decisions and consequences as percent of Model I's decisions and consequences									
Optimal Decisions	G_r^*	100.00%	99.48%	101.84%	100.99%	100.00%	150.00%	150.00%	151.94%
	G_m^*	100.00%	100.97%	99.04%	101.01%	100.00%	NA	NA	NA
	Q_r^*	100.00%	99.78%	99.56%	99.01%	131.66%	122.47%	140.36%	139.28%
	K^*	100.00%	100.00%	100.00%	100.00%	66.67%	133.33%	100.00%	100.00%
Optimal Consequences	P_r	100.00%	100.16%	100.31%	100.70%	100.00%	82.43%	82.43%	83.12%
	P_m	100.00%	100.66%	99.34%	100.69%	100.00%	NA	NA	NA
	D	100.00%	99.56%	99.12%	98.00%	100.00%	150.00%	150.00%	148.06%
	Π_m	100.00%	NA	96.53%	NA	100.00%	NA	NA	NA
	Π_r	100.00%	98.74%	NA	NA	100.00%	NA	NA	NA
	Π	100.00%	99.65%	99.32%	98.42%	100.00%	119.38%	119.38%	NA
	C_m	100.00%	NA	99.66%	NA	NA	114.24%	NA	NA
	C_r	100.00%	99.78%	NA	NA	NA	122.47%	NA	NA
	C	100.00%	99.80%	99.61%	99.10%	99.16%	118.37%	118.05%	NA
	Z_m	100.00%	101.13%	96.13%	97.93%	NA	NA	NA	NA
	Z_r	100.00%	98.65%	101.38%	98.62%	NA	NA	NA	NA
	Z	100.00%	99.64%	99.29%	98.35%	100.09%	119.48%	119.52%	119.55%

Table 2: Optimized decisions and results for various models assuming $e = 110\%$ of base case

Assumptions common to all models Same as base case, except $e = 4,400$ units/dollar									
Part 1: Optimized decisions and consequences									
	Variable	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
Optimal Decisions	G_r^*	\$2.87	\$2.85	\$2.94	\$2.90	\$2.87	\$4.31	\$4.31	\$4.37
	G_m^*	\$2.87	\$2.90	\$2.84	\$2.90	\$2.87	\$0.00	\$0.00	\$0.00
	Q_r^*	2,513.30	2,506.23	2,498.26	2,483.40	3,308.91	3,078.15	3,518.10	3,486.84
	K^*	3.00	3.00	3.00	3.00	2.00	4.00	3.00	3.00
Optimal Consequences	P_r	\$8.49	\$8.51	\$8.53	\$8.56	\$8.49	\$7.06	\$7.06	\$7.12
	P_m	\$4.37	\$4.40	\$4.34	\$4.40	\$4.37	\$1.50	\$1.50	\$1.50
	D	12,633.33	12,562.38	12,482.60	12,339.55	12,633.33	18,950.00	18,950.00	18,666.92
	Π_m	\$16,272.98	NA	\$15,412.37	NA	\$16,272.98	NA	NA	NA
	Π_r	\$26,272.98	\$25,860.12	NA	NA	\$26,272.98	NA	NA	NA
	Π	\$42,545.96	\$42,341.08	\$42,108.01	\$41,682.83	\$42,545.96	\$51,614.20	\$51,614.20	NA
	C_m	\$2,500.03	NA	\$2,488.21	NA	NA	\$2,875.50	NA	NA
	C_r	\$2,513.30	\$2,506.23	NA	NA	NA	\$3,078.15	NA	NA
	C	\$5,013.33	\$5,000.71	\$4,986.47	\$4,960.88	\$4,963.37	\$5,953.65	\$5,925.07	NA
	Z_m	\$13,772.95	\$13,986.48	\$12,924.16	\$13,363.87	NA	NA	NA	NA
	Z_r	\$23,759.68	\$23,353.89	\$24,197.38	\$23,358.08	NA	NA	NA	NA
Z	\$37,532.63	\$37,340.37	\$37,121.54	\$36,721.95	\$37,582.59	\$45,660.55	\$45,689.13	\$45,707.11	
Part 2: Percent changes in decisions and consequences as a result of a 10% increase in e (i.e., changes from the decision and consequences in Table 1)									
Optimal Decisions	G_r^*	-11.66%	-11.71%	-11.18%	-11.50%	-11.66%	-11.66%	-11.66%	-11.48%
	G_m^*	-11.66%	-11.50%	-11.86%	-11.52%	-11.66%	NA	NA	NA
	Q_r^*	-1.42%	-1.48%	-1.58%	-1.62%	-1.42%	-1.42%	-1.69%	-1.81%
	K^*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Optimal Consequences	P_r	-8.19%	-8.16%	-8.10%	-8.11%	-8.19%	-7.45%	-7.45%	-7.37%
	P_m	-7.97%	-7.89%	-8.09%	-7.91%	-7.97%	NA	NA	NA
	D	-2.82%	-2.94%	-3.13%	-3.14%	-2.82%	-2.82%	-2.82%	-3.02%
	Π_m	-26.86%	NA	-28.24%	NA	-26.86%	NA	NA	NA
	Π_r	-18.53%	-18.79%	NA	NA	-18.53%	NA	NA	NA
	Π	-21.93%	-22.04%	-22.21%	-22.29%	-21.93%	-20.67%	-20.67%	NA
	C_m	-1.12%	NA	-1.25%	NA	NA	-0.44%	NA	NA
	C_r	-1.42%	-1.48%	NA	NA	NA	-1.42%	NA	NA
	C	-1.27%	-1.32%	-1.41%	-1.42%	-1.42%	-0.95%	-1.15%	NA
	Z_m	-30.16%	-29.88%	-31.83%	-30.81%	NA	NA	NA	NA
	Z_r	-20.00%	-20.29%	-19.64%	-20.26%	NA	NA	NA	NA
Z	-24.06%	-24.17%	-24.35%	-24.45%	-24.02%	-22.68%	-22.65%	-22.64%	

Table 3: An analysis of the supply chain's costs of a 10% misestimate in the value of e

Assumptions common to all models									
Same as the base case, except that e is assumed to be 4,000 units/dollar when its true value is 4,400 units/dollar									
Part 1: Optimized decisions under incorrectly assumed value of e and the corresponding true consequences									
	Variable	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
Decisions	G_r^*	\$3.25	\$3.23	\$3.31	\$3.28	\$3.25	\$4.88	\$4.88	\$4.94
	G_m^*	\$3.25	\$3.28	\$3.22	\$3.28	\$3.25	\$0.00	\$0.00	\$0.00
	Q_r^*	2,550	2,544	2,538	2,524	3,357	3,122	3,579	3,551
	K^*	3	3	3	3	2	4	3	3
True Consequences	P_r	\$9.25	\$9.26	\$9.28	\$9.32	\$9.25	\$7.63	\$7.63	\$7.69
	P_m	\$4.75	\$4.78	\$4.72	\$4.78	\$4.75	\$1.50	\$1.50	\$1.50
	D	9,300	9,237	9,175	9,014	9,300	16,450	16,450	16,174
	Π_m	\$10,225.00	NA	\$9,531.25	NA	\$10,225.00	NA	NA	NA
	Π_r	\$20,225.00	\$19,861.77	NA	NA	\$20,225.00	NA	NA	NA
	Π	\$30,450.00	\$30,170.81	\$29,897.45	\$29,176.56	\$30,450.00	\$50,193.75	\$50,193.75	NA
	C_m	\$2,189.63	NA	\$2,159.45	NA	NA	\$2,802.84	NA	NA
	C_r	\$2,158.67	\$2,179.66	NA	NA	NA	\$2,878.30	NA	NA
	C	\$4,348.29	\$4,345.06	\$4,332.24	\$4,298.66	\$4,318.38	\$5,681.15	\$5,616.25	NA
	Z_m	\$8,035.37	\$8,143.64	\$7,371.80	\$7,447.02	\$8,277.58	NA	NA	NA
	Z_r	\$18,066.33	\$17,682.12	\$18,193.41	\$17,430.88	\$17,854.04	NA	NA	NA
Z	\$26,101.71	\$25,825.75	\$25,565.21	\$24,877.90	\$26,131.62	\$44,512.60	\$44,577.50	\$44,285.71	
Part 2: Percent change in each model's actual decisions and consequences from the optimal decisions and consequences (i.e., from the decisions and consequences in Table 2)									
Decisions	G_r^*	13.19%	13.26%	12.59%	13.00%	13.19%	13.19%	13.19%	12.97%
	G_m^*	13.19%	13.00%	13.46%	13.02%	13.19%	NA	NA	NA
	Q_r^*	1.44%	1.50%	1.60%	1.65%	1.44%	1.44%	1.72%	1.84%
	K^*	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Consequences	P_r	8.92%	8.88%	8.82%	8.83%	8.92%	8.05%	8.05%	7.95%
	P_m	8.67%	8.57%	8.80%	8.59%	8.67%	NA	NA	NA
	D	-26.39%	-26.47%	-26.50%	-26.95%	-26.39%	-13.19%	-13.19%	-13.35%
	Π_m	-37.17%	NA	-38.16%	NA	-37.17%	NA	NA	NA
	Π_r	-23.02%	-23.20%	NA	NA	-23.02%	NA	NA	NA
	Π	-28.43%	-28.74%	-29.00%	-30.00%	-28.43%	-2.75%	-2.75%	NA
	C_m	-12.42%	NA	-13.21%	NA	NA	-2.53%	NA	NA
	C_r	-14.11%	-13.03%	NA	NA	NA	-6.49%	NA	NA
	C	-13.27%	-13.11%	-13.12%	-13.35%	-12.99%	-4.58%	-5.21%	NA
	Z_m	-41.66%	-41.77%	-42.96%	-44.27%	NA	NA	NA	NA
	Z_r	-23.96%	-24.29%	-24.81%	-25.38%	NA	NA	NA	NA
Z	-30.46%	-30.84%	-31.13%	-32.25%	-30.47%	-2.51%	-2.43%	-3.11%	