

**A Dual Pricing Model For Price Sensitive Products Subject To Sudden  
Obsolescence**

Patrick Lee, Fairfield University  
Prafulla Joglekar, La Salle University

**Forthcoming in**  
*Journal of Business and Economic Studies*, Fall 2005

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### **Abstract**

For the past five decades, researchers have investigated the relationship between a firm's pricing and inventory decisions by combining the concepts of the economic order quantity theory with those of the price theory. Ever since the earliest works on this topic, it has been assumed that when a product's unit cost is constant and its demand curve is known and stationary, a retailer of the product would find it optimal to buy a fixed quantity every time he buys and to sell the product at a single price throughout the year.

However, when inventory cost considerations are to be combined with the considerations of a price sensitive demand curve, one must keep in mind that, once an order quantity has arrived in the retailer's stock, the inventory on hand is a declining function of time. Consequently, during any inventory cycle, the retailer's incremental inventory carrying costs are also a declining function of time. Thus, it seems appropriate that a retailer should set different selling prices at different points in time during an inventory cycle. This would be particularly true for products that can become obsolete overnight. Clearly, early in an inventory cycle, a greater amount of inventory is at risk of being obsolete than the amount at risk towards the end of the cycle. Thus, in comparison to the price towards the end of an inventory cycle, in the early part of the cycle, a retailer should set a lower selling price so as to move the inventory faster.

In theory, one can visualize a system where, within an inventory cycle, price is a continuous function of the time elapsed from the beginning of the cycle. However, such a system would not be practical since it would involve price changes in fractional pennies from one unit to the next. On the other hand, Joglekar (2003) has shown that a dual pricing system – involving two different prices over two different periods within each inventory cycle – is feasible and more profitable for the retailer. However, Joglekar's (2003) model did not consider products that can become obsolete overnight. In this paper, we present a model that extends Joglekar's (2003) dual pricing approach to the situation of products subject to sudden obsolescence. Our model demonstrates that the dual pricing strategy always produces a profit that is greater than or equal to the profit produced by the popular single pricing strategy. Sensitivity analysis shows that the greater the price elasticity and the shorter the expected life of the product, the more profitable the dual pricing strategy is.

### **INTRODUCTION**

Classical price theory tells us that, when a product's demand is price sensitive but the demand curve is known and stationary, the retailer's optimal strategy is to charge a single price throughout the year. However, the classical price theory simply ignores inventory

related costs. Whitin (1955) was the first one to combine the concepts from the economic order quantity (EOQ) theory with the concepts from the price theory to investigate the simultaneous determination of price and order quantity decisions of a retailer. Whitin (1955) assumed that, when all the other assumptions of the EOQ model are valid but demand is price sensitive, with a known and stationary demand curve, a retailer's optimal strategy would be, once again, to buy a fixed quantity every time he buys and to sell it at a single price throughout the inventory cycle. Kunreuther and Richard (1971) built similar models for determining the optimal price and order quantity for a retailer as well as a manufacturer. They also assumed that the optimal strategy would involve a single price throughout the year and a particular order quantity for each order.

Over the years, Whitin's (1955) and Kunreuther and Richard's (1971) works have served as foundations to a number of other works (Tersine and Price 1981; Arcelus and Srinivasan 1987; Ardalan 1991; Hall 1992; Martin 1994; Arcelus and Srinivasan 1998; and Abad 2003). None of these authors questioned Whitin's (1955) and Kunreuther and Richard's (1971) assumption that the retailer's optimal strategy would be to sell the product at a single price throughout the year. However, in a recent paper, Joglekar (2003) has shown that a dual pricing system – involving two different prices over two different periods within each inventory cycle – is feasible and more profitable for the retailer. Joglekar's (2003) argument is that during an inventory cycle, a retailer's inventory level and carrying costs are a declining function of time. When a retailer faces a constant demand regardless of his selling price, there is nothing he can do about these changes in his carrying costs within an inventory cycle. As such, the retailer's optimal strategy is to charge a single price throughout an inventory cycle. However, when his demand is price sensitive, the retailer can adopt a time-dependent pricing strategy to minimize the impact of his time-dependent inventory carrying costs. The idea is to charge a lower selling price when on-hand inventory is large. The lower price generates a higher demand, depleting that inventory rapidly. Later in the inventory cycle, when the inventory on hand is low, it is not important to reduce it as rapidly and the retailer can set a higher selling price.

Joglekar (2003) points out that, in theory, the retailer would maximize his annual profit by making his selling price a continuous function of his on-hand inventory. On the other hand, a continuously changing price would not be practical in real life since it would involve price changes from each unit of sale to the next and changes that are in fractions of pennies. Hence, Joglekar (2003) considers a dual pricing system involving a relatively smaller selling price in the first part of an inventory cycle followed by a higher price in the later part of the cycle. He finds that, for a retailer, a dual pricing strategy always produced a profit that was greater than or equal to the profit produced by a single price strategy. Of course, in Joglekar's (2003) numerical examples, the difference between the two profits is modest. However, Joglekar (2003) did not consider products that can become obsolete overnight. We suspect that when a product is subject to such obsolescence, the dual pricing strategy would be substantially more profitable. This is because early in an inventory cycle, a greater amount of inventory is at risk of being obsolete than the amount at risk towards the end of the cycle. Hence, in this paper, we present a model that extends Joglekar's (2003) dual pricing approach to the situation of products subject to sudden obsolescence.

For prudent order quantity and pricing decisions on products subject to sudden obsolescence, a retailer must account for the costs of obsolescence carefully. Inventory theory's traditional way of accounting for the obsolescence costs is to incorporate them as a component of the holding costs in the EOQ model (Hadley and Whitin 1963; Naddor 1966; Silver and Peterson 1985). Of course, Hadley and Whitin (1963) did warn that it was inappropriate to treat obsolescence costs as a mere component of inventory carrying costs. Masters (1991) identified several models (e.g., Barbosa and Friedman 1979; Brown 1982; Hill, Girard, and Mabert 1989) that treated obsolescence costs separately from other inventory carrying costs. However, he pointed out that these models dealt with *gradual* rather than *sudden* obsolescence. Masters defined *sudden obsolescence* as a situation when a product's lifetime is negative exponentially distributed, and consequently, the probability of obsolescence is constant until the obsolescence actually occurs. Masters concluded that for products subject to sudden obsolescence, the use of the EOQ model was appropriate, provided that the obsolescence component was computed properly. Masters' (1991) model showed that for a product with zero salvage value, the proper obsolescence cost (in percent terms) was the reciprocal of the product's expected life. In other words, for a product with expected life of two years, the obsolescence cost would be 50% per year of the value of the inventory. Yet, as Joglekar and Lee (1993) have pointed out, current industry practice seriously underestimates the cost of sudden obsolescence. The common practice is to use an estimate of 1 to 3% per year for obsolescence costs. Note that by Masters' formula, an obsolescence component on the order of 1% is appropriate only if the product has an expected life of 100 years!

While Masters used an approximate model, Joglekar and Lee (1993) built a formula to exactly account for all the costs of sudden obsolescence. They showed that Masters' model also underestimated the true lifetime costs of a stocking policy and that the associated errors were substantial particularly in the cases of very short expected lives. Joglekar and Lee (1996) extended Joglekar and Lee's (1993) work by considering a product's expected lifetime profit from a retailer's stocking decisions on products subject to sudden obsolescence. In this paper, we use Joglekar and Lee's (1993 and 1996) approach of accounting for the exact costs of sudden obsolescence and a product's expected lifetime profit to assess the merits of a dual pricing strategy compared to a single price strategy.

## THE NOTATION

We shall set up two models- one for the single price strategy and one for the dual price strategy. The notation common to both the models is as below:

- $C$  = Retailer's unit cost of buying the product
- $C_o$  = Retailer's ordering cost per order
- $V$  = Retailer's selling and other variable costs per unit
- $H$  = Retailer's carrying costs per dollar of inventory per year
- $S_o$  = Retailer's salvage value per unit of inventory left over at obsolescence
- $L$  = expected lifetime of the product

$Q$  = Total ordering quantity per order

$Q = Q_1 + Q_2$  in the dual pricing model

$Q_1$  = Quantity sold at  $S_{p1}$  per unit during the first part of an inventory cycle in the dual pricing model

$Q_2$  = Quantity sold at  $S_{p2}$  per unit during the latter part of an inventory cycle in the dual pricing model

$T$  = the inventory cycle time

$T = Q/R$  in the single price model

$T = T_1 + T_2 = Q_1/R_1 + Q_2/R_2$  in the dual pricing model

$T_1$  = the time it takes to deplete  $Q_1$  units of inventory at the selling price of  $S_{p1}$

$T_1 = Q_1/R_1$

$T_2$  = the time it takes to deplete  $Q_2$  units of inventory at the selling price of  $S_{p2}$

$T_2 = Q_2/R_2$

$S_p$  = Retailer's selling price per unit in the single price model

$S_{p1}$  = Retailer's selling price per unit during the first part of an inventory cycle in the dual pricing model

$S_{p2}$  = Retailer's selling price per unit during the latter part of an inventory cycle in the dual pricing model

$R(S_p) = R$  = Retailer's annual demand as a declining function of selling price,  $S_p$ .

$R(S_p)$  is a known and stationary function.

$R(S_{p1}) = R_1$  = Retailer's annual demand as a function of  $S_{p1}$

$R(S_{p2}) = R_2$  = Retailer's annual demand as a function of  $S_{p2}$

$K$  = Probability that the product would not become obsolete during an entire inventory cycle

$$K = \int_T^{\infty} \frac{1}{L} e^{-(t/L)} dt = e^{-\frac{T}{L}} \text{ in single price model.}$$

$$K = e^{-\frac{T_1+T_2}{L}} = e^{-\frac{T}{L}} \text{ in the dual pricing model.}$$

$K_1$  = Probability that the product would not become obsolete during the first part of an inventory cycle in the dual pricing model

$$K_1 = \int_{T_1}^{\infty} \frac{1}{L} e^{-(t/L)} dt = e^{-\frac{T_1}{L}}.$$

$K_2$  = Probability that the product would not become obsolete during the latter part of an inventory cycle in the dual pricing model

$$K_2 = \int_{T_2}^{\infty} \frac{1}{L} e^{-(t/L)} dt = e^{-\frac{T_2}{L}}.$$

$E[P_c]$  = Retailer's profit from one inventory cycle, as expected at the beginning of an inventory cycle

$E[P_c]$  is a function of  $Q$  and  $S_p$  in the single price model

$E[P_c]$  is a function of  $Q_1$ ,  $S_{p1}$ ,  $Q_2$  and  $S_{p2}$  in the dual pricing model

$E[P_L]$  = Retailer's profit from the product's lifetime, as expected at the beginning of an inventory cycle

$E[P_L]$  is a function of  $Q$  and  $S_p$  in the single price model

$E[P_L]$  is a function of  $Q_1$ ,  $S_{p1}$ ,  $Q_2$  and  $S_{p2}$  in the dual pricing model

## THE SINGLE PRICE MODEL

In this model we assume that a retailer buys  $Q$  units at the beginning of an inventory cycle and plans to charge a single price  $S_p$  for those units until obsolescence occurs. If and when obsolescence occurs, all remaining units would be sold at the salvage value of  $S_o$ .

The retailer's cost of acquiring an order quantity  $Q$  at the beginning of an inventory cycle is given by

$$(1) \quad CQ + C_o$$

If obsolescence occurs at time  $0 \leq t < T$ , only  $tR$  units sell at  $S_p$ . The remaining  $(Q - tR)$  units sell at  $S_o$ . Hence, the retailer's revenue net of his variable selling costs is given by

$$(2) \quad \int_0^T \{[tR(S_p - V) + (Q - tR)(S_o)]\} \frac{1}{L} e^{-\frac{t}{L}} dt = R(S_p - V - S_o)[L(1 - K) - TK] + Q S_o(1 - K)$$

In this situation, the retailer's inventory carrying costs are:

$$(3) \quad \int_0^T \{[Q - (\frac{tR}{2})]CH\} \frac{1}{L} e^{-\frac{t}{L}} dt = \{Q[L(1 - K) - TK] - \frac{R}{2}[2L^2(1 - K) - (2L + T)TK]\}CH$$

On the other hand, if the product becomes obsolete at  $t \geq T$ , the retailer's revenue net of variable selling costs is given by

$$(4) \quad \int [R(S_p - V)T] \frac{1}{L} e^{-\frac{t}{L}} dt = R(S_p - V)TK$$

In this situation, the retailer's inventory carrying costs are:

$$(5) \quad \int_T^{\infty} (\frac{Q}{2}TCH) \frac{1}{L} e^{-\frac{t}{L}} dt = (\frac{Q}{2}TCH)K$$

Thus, the retailer's profit, as expected at the beginning of an inventory cycle, is given by

$$(6) \quad E[P_c] = (2) - (3) + (4) - (5) - (1)$$

Given a constant obsolescence rate, and a time-invariant order quantity, unless obsolescence has occurred in the previous inventory cycle, the retailer's profit from the lifetime of a product as expected at the beginning of an order cycle is identical to that expected at the beginning of the previous order cycle. Hence, As Joglekar and Lee (1996) have shown,

$$(7) \quad E[P_L] = E[P_c]/(1 - K)$$

In the single price model, we want to maximize this profit with respect to  $Q$  and  $S_p$ . While we could derive the first order and second order conditions for this maximization, we find that in today's environment that kind of derivation is unnecessary. Instead, in this

paper, we use Excel's Solver facility to obtain our optimal solution. Unlike calculus-based solutions, the use of Solver enables us to impose such practical requirements as order quantities must be in integer units and all prices when converted to pennies must be integer. Of course, we also need to specify non-negativity constraints for the ordering quantities, demands, and selling prices.

### THE DUAL PRICING MODEL

In the dual pricing model, we assume that a retailer buys  $Q$  ( $=Q_1 + Q_2$ ) units at the beginning of an inventory cycle and plans to charge a price  $S_{p1}$  for the first  $Q_1$  units (i.e., until time  $T_1$ ) followed by a price of  $S_{p2}$  for the remaining  $Q_2$  units. Of course, if and when obsolescence occurs, all remaining units would be sold at the salvage value of  $S_o$ .

As in the single price model, at the beginning of an inventory cycle, the retailer incurs the cost of acquiring an order quantity of  $Q$  units which is given by

$$(8) \quad CQ + C_o$$

If obsolescence occurs at time  $t$ ,  $0 \leq t < T_1$ , only  $tR_1$  units sell at  $S_{p1}$ , and the remaining  $(Q - tR_1)$  units sell at  $S_o$ . Hence, the expected revenue net of variable selling costs is given by

$$(9) \quad \int_0^{T_1} \{[tR_1(S_{p1} - V) + (Q - tR_1)(S_o)]\} \frac{1}{L} e^{-\frac{t}{L}} dt = R_1(S_{p1} - V - S_o)[L(1 - K_1) - T_1 K_1] + Q S_o(1 - K_1)$$

The corresponding expected inventory carrying costs are

$$(10) \quad \int_0^{T_1} \{(Q - \frac{tR_1}{2})CHt\} \frac{1}{L} e^{-\frac{t}{L}} dt = \{Q[L(1 - K_1) - T_1 K_1] - \frac{R_1}{2}[2L^2(1 - K_1) - (2L + T_1)T_1 K_1]\}CH$$

If obsolescence occurs at time  $T_1 \leq t < T_1 + T_2$ , then  $T_1 R_1$  units sell at  $S_{p1}$ ,  $(t - T_1)R_2$  units sell at  $S_{p2}$ , and the remaining units sell at  $S_o$ . Hence, the retailer's expected revenue net of variable selling costs is given by

$$(11) \quad \int_{T_1}^{T_1+T_2} \{T_1 R_1(S_{p1} - V) + (t - T_1)R_2(S_{p2} - V) + [Q_2 - R_2(t - T_1)](S_o)\} \frac{1}{L} e^{-\frac{t}{L}} dt$$

$$= \{T_1 R_1(S_{p1} - V) - T_1 R_2(S_{p2} - V) + Q_2 S_o + R_2 S_o T_1\} [K_1(1 - K_2)]$$

$$+ [R_2(S_{p2} - V - S_o)][(L + T_1)K_1(1 - K_2) - T_2 K_1]$$

The corresponding inventory holding costs are

$$(12) \quad \int_{T_1}^{T_1+T_2} \{[T_1(\frac{Q_1}{2} + Q_2)] + [Q_2 - R_2(\frac{t - T_1}{2})(t - T_1)]\} CH \frac{1}{L} e^{-\frac{t}{L}} dt$$

$$= CH\left\{T_1\left(\frac{Q_1}{2}+Q_2\right)-Q_2T_1-\frac{R_2}{2}T_1^2\right\}(K_1)(1-K_2)+\left(Q_2+R_2T_1\right)\left[\left(L+T_1\right)K_1(1-K_2)-T_2K\right]$$

$$-\frac{R_2}{2}\left[2L^2K_1(1-K_2)+2L\left(T_1K_1-TK\right)+T_1^2K_1-T^2K\right]$$

If obsolescence occurs at  $t \geq T_1 + T_2$ ,  $Q_1$  units sell at  $S_{p1}$ , and  $Q_2$  units sell at  $S_{p2}$ . Hence, the retailer's expected revenue net of variable selling costs is given by

$$(13) \quad \int_T^{\infty} \left\{T_1R_1(S_{p1}-V)+T_2R_2(S_{p2}-V)\right\} \frac{1}{L} e^{-\frac{t}{L}} dt = [T_1R_1(S_{p1}-V)+T_2R_2(S_{p2}-V)]K$$

The corresponding inventory holding costs are

$$(14) \quad \int_T^{\infty} \left[ \frac{(Q_1+Q_2)+Q_2}{2} T_1 + \frac{Q_2}{2} T_2 \right] CH \frac{1}{L} e^{-\frac{t}{L}} dt = \left[ \left( \frac{Q_1}{2} + Q_2 \right) T_1 + \frac{Q_2}{2} T_2 \right] CHK$$

Thus, in the dual price model, the retailer's net profit during one inventory cycle of  $Q$  units ( $T_1 + T_2$  years), as expected at the beginning of that cycle is given by

$$(15) \quad E[P_c] = (9) - (10) + (11) - (12) + (13) - (14) - (8)$$

As in the single price model, the corresponding lifetime profit is given by

$$(16) \quad E[P_L] = E[P_c]/(1-K)$$

To solve this model, we want to maximize (16) with respect to the four variables  $Q_1$ ,  $Q_2$ ,  $S_{p1}$ , and  $S_{p2}$ . A calculus based optimization approach here would be daunting. Notice that the functions are not convex, and we can only obtain local optima provided that they exist. However, we find it unnecessary to pursue the calculus-based approach. We shall simply use Excel Solver to accomplish the task. As mentioned before an advantage of using Solver is that we can accommodate such practical requirements as order quantities must be in non-negative integer units and all prices when converted to pennies must be non-negative integers. Finally, we also need to specify non-negativity constraints for demands.

## NUMERICAL EXAMPLES

In what follows, we present a numerical comparison of the decisions and consequences resulting from the two pricing policies for a base case example. In addition, we also provide an analysis of the sensitivity of the consequences to the assumed values of the parameters.

Table 1 presents our base case numerical example as well as sensitivity analysis. In the base case, we assume that the retailer purchases a product at \$8/unit. His ordering cost is \$1,000/order. Actually, although we call this cost as the ordering cost, a portion of it is the cost of advertising, promotion and other administrative costs that the retailer would

incur during any inventory cycle. We believe that unless a product is discontinued, at the beginning of an inventory cycle, a retailer must commit to spend a fixed amount for these types of expenses during that inventory cycle. Thus, those expenses can be grouped with the actual ordering costs. \$1,000 per order cycle is certainly a reasonable numerical value for these costs of ordering, advertising, promotion and other administrative support for the product. The retailer's inventory carrying cost is \$0.30 per dollar of inventory per year. The retailer incurs a selling cost of \$5/unit for each unit sold before obsolescence. The selling costs are not incurred once the product becomes obsolete. At obsolescence, all leftover stock is liquidated at the salvage value of \$1/unit. The product's obsolescence rate is constant and expected life is 1 year. Finally, product demand is a linearly declining function of the retail price, given by  $R = 100,000 - 5,500S_p$ .

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 Insert Table 1 about here  
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As can be seen from Table 1, the optimal order quantity in the single price model is 1,601 units, and the optimal retail price is \$15.90/unit. Of course, whether the retailer adopts the single price strategy or the dual price strategy, whenever obsolescence occurs, the remaining stock will be sold at the salvage value of \$1/unit. The single price strategy leads to an expected profit of \$2,438.58/cycle. The product's lifetime profit is \$20,360.88. In comparison, in the dual price model, the optimal order quantity ( $Q_1 + Q_2$ ) is 1,626 units per order. Of that total order quantity, the retailer sells the first 879 units at a retail price of \$15.75/unit and the remaining 747 units at a retail price of \$16.07/unit. The resultant expected profit is 2,500.09 per inventory cycle. This represents a 2.52% increase in per cycle compared to the single price strategy. However, a comparison of the per cycle profit of the two strategies is unfair since the cycle lengths of the two strategies are different. Hence, we compare the expected lifetime profits of the two strategies. The dual price strategy results in a lifetime profit of \$20,503.72. This lifetime profit still represents a 0.70% increase compared to the lifetime profit resulting from the single price strategy. While this increase in expected profit is modest, clearly, the dual price strategy is better than the single price strategy. This result is consistent with Joglekar's (2003) finding.

In Table 1, in addition to the base case decisions and results, we present an analysis of the sensitivity of those results to the various parameters of the model. For each parameter, we present two sets of results, one assuming a parameter value lower than its base case value and the other assuming a parameter value higher than its base case value. As can be seen there, our results are most sensitive to the assumed elasticity of demand. As the elasticity increases, the dual price strategy's profit in relation to the profit of the single price strategy increases substantially. Keeping all other parameter values of the base case, when the demand function is  $R = 100,000 - 6,000S_p$ , the profit from the dual price strategy is 3.97% higher than the profit from the single price strategy.

Given this discussion, the rest of the numbers in Table 1 should be self-explanatory. It should be clear from Table 1 that the expected lifetime profit from the dual price strategy is always greater than or equal to that from the single price strategy. This result is also consistent with Joglekar's (2003) findings. Also observe that among the remaining

parameters, in order of their declining impact, our results are most sensitive to the assumed unit cost, ordering cost/order, selling cost/unit, expected life, salvage value/unit, and holding cost/dollar/year. Clearly, if a product's demand is highly price sensitive, and simultaneously its unit cost, ordering costs, and selling costs are high while its expected life and salvage value is small, the dual price strategy would prove to be the most advantageous compared to the single price strategy.

## CONCLUSION

Joglekar (2003) argued that during an inventory cycle, a retailer's inventory level and carrying costs are a declining function of time. Hence, a time-variant pricing strategy is preferable compared to a single price strategy assumed by prior researchers studying determination of price and order quantity decisions simultaneously. In theory, one can visualize a system where, within an inventory cycle, price is a continuous function of the time elapsed from the beginning of the cycle. However, such a system would not be practical since it would involve price changes in fractional pennies from one unit to the next. On the other hand, Joglekar (2003) showed that a dual pricing system – involving two different prices over two different periods within each inventory cycle – is feasible and more profitable for the retailer. Our model shows that Joglekar's (2003) findings are also valid in the case of a product subject to sudden obsolescence. When a product's demand is highly price sensitive, its expected life is short (often less than a year), and its salvage value is small or negative (as is sometimes the case), it would be particularly important for a retailer to pursue the dual price strategy.

## REFERENCES

- Abad P. (2003). Optimal price and lot size when the supplier offers a temporary price reduction over an interval. *Computers and Operations Research*, 30, 63-74.
- Arcelus, F. and G. Srinivasan. (1987). Inventory policies under various optimizing criteria and variable markup rate. *Management Science*, 33, 756-762.
- Arcelus, F. and G. Srinivasan. (1998). Ordering policies under one time only discount and price sensitive demand. *IIE Transactions*, 30, 1057-64.
- Ardalan, A. (1991). Combined optimal price and optimal inventory replenishment policies when a sale results in increase in demand. *Computers and Operations Research*, 18, 721-730.
- Barbosa, L. and M. Friedman. (1979). Inventory lot size models with vanishing markets. *Journal of the Operations Research Society*, 30(2), 1129-1132.
- Brown, R. (1982). *Advanced service parts management*. Materials Management Systems, Inc., Norwich, VT.
- Hall, R. (1992). Price changes and order quantities: impacts of discount rate and storage costs, *IIE Transactions*, 24, 104-10.
- Hadley, G. and T. Whitin. (1963). *Analysis of inventory systems*.
- Hill, A., V. Girard and V. Mabert. (1989). A decision support system for determining optimal retention stocks for service parts inventories. *IIE Transactions*, 21(3), 236-248.

- Joglekar, P. (2003). Optimal price and order quantity strategies for the retailer of a product with price sensitive demand. *Proceedings of Academy of information and management sciences, Tunica Meeting*, 7(1), 13-19.
- Joglekar P. and P. Lee. (1996). A profit maximization model for a retailer's stocking decisions on products subject to sudden obsolescence. *Production and Operations Management* 5(3), 288-294.
- Joglekar P. and P. Lee. (1993). An exact formulation of inventory costs and optimal lot size in face of sudden obsolescence. *Operations Research Letters*, 14, 283-290.
- Kunreuther, H. and J. Richard. (1971). Optimal pricing and inventory decisions for nonseasonal items. *Econometrica*, 39, 173-175.
- Lambert, D. and B. LaLonde. (1976). Inventory carrying costs. *Management Accounting*, 58, 31-35.
- Martin, G. (1994). Note on an EOQ model with a temporary sale price. *International Journal of Production Economics*, 37, 241-243.
- Masters, J. (1991). A note on the effect of sudden obsolescence on the optimal lot size. *Decision Sciences*, 22, 1180-1186.
- Naddor, E. (1966). *Inventory systems*.
- Silver, E. and R. Peterson. (1985). *Decision systems for inventory management and production planning*.
- Tersine, R. (1988). *Principles of inventory and materials management*.
- Tersine, R. and R. Price. (1981). Temporary price discounts and EOQ. *Journal of Purchasing and Materials Management*, 17, 23-27.
- Whitin, T. (1955). Inventory control and price theory. *Management Science*, 2, 61-68.

**Table 1**  
**Numerical Example & Sensitivity Analysis**

Base Case Assumptions											
$C = \$8/\text{unit}$ . $C_o = \$1,000/\text{order}$ . $H = \$0.30/\$ \text{ inventory/year}$ , $V = \$5/\text{unit}$ . $S_o = \$1/\text{unit}$ . $L = 1 \text{ year}$ . $R = 100,000 - 5,500S_p$ .											
Changed Assumptions (if any)	Results										
	Single Price Model				Dual Price Model						% increase in $E(P_L)$ due to Dual Pricing
	Decisions		Consequences		Decisions				Consequences		
	$Q$	$S_p$	$E(P_c)$	$E(P_L)$	$Q_1$	$Q_2$	$S_{p1}$	$S_{p2}$	$E(P_c)$	$E(P_L)$	
Base Case (No changes)	1601	\$15.90	\$2,438.58	\$20,360.88	879	747	\$15.75	\$16.07	\$2,500.09	\$20,503.72	
$R = 100,000 - 5,000S_p$	1819	\$16.77	\$4,557.77	\$42,787.86	975	862	\$16.64	\$16.92	\$4,621.11	\$42,887.02	0.23%
$R = 100,000 - 6,000S_p$	1335	\$15.20	\$819.57	\$5,822.52	795	575	\$15.03	\$15.43	\$887.11	\$6,053.90	3.97%
$L=0.5 \text{ year}$	1170	\$16.00	\$1,311.51	\$7,402.75	666	529	\$15.80	\$16.26	\$1,377.93	\$7,541.22	1.87%
$L=2 \text{ years}$	2066	\$15.83	\$3,671.36	\$47,832.04	1103	989	\$15.71	\$15.97	\$3,744.20	\$47,999.30	0.35%
$C = \$7/\text{unit}$	1921	\$15.35	\$4,132.99	\$35,618.28	1030	913	\$15.22	\$15.49	\$4,193.57	\$35,716.52	0.28%
$C = \$9/\text{unit}$	1296	\$16.47	\$1,078.47	\$8,386.30	758	571	\$16.29	\$16.70	\$1,146.35	\$8,608.28	2.65%
$C_o = \$500/\text{order}$	1162	\$15.80	\$2,143.05	\$25,247.34	622	553	\$15.70	\$15.92	\$2,179.21	\$25,313.72	0.26%
$C_o = \$2,000/\text{order}$	2170	\$16.04	\$2,256.33	\$13,411.42	1264	959	\$15.83	\$16.32	\$2,391.84	\$13,732.42	2.39%
$V = \$4/\text{unit}$	1781	\$15.37	\$3,741.23	\$34,392.80	953	848	\$15.23	\$15.52	\$3,798.55	\$34,507.13	0.33%
$V = \$6/\text{unit}$	1392	\$16.44	\$1,255.04	\$9,280.13	795	630	\$16.27	\$16.65	\$1,322.57	\$9,472.47	2.07%
$S_o = \$0/\text{unit}$	1517	\$15.91	\$2,236.91	\$19,565.70	844	699	\$15.76	\$16.10	\$2,307.45	\$19,724.59	0.81%
$S_o = \$2/\text{unit}$	1697	\$15.88	\$2,659.87	\$21,202.83	938	785	\$15.74	\$16.05	\$2,727.03	\$21,329.81	0.60%
$H = \$0.20/\$ \text{ year}$	1678	\$15.88	\$2,610.60	\$21,030.32	917	785	\$15.74	\$16.05	\$2,676.50	\$21,160.50	0.62%
$H = \$0.40/\$ \text{ year}$	1533	\$15.91	\$2,277.06	\$19,721.41	838	720	\$15.75	\$16.09	\$2,339.74	\$19,876.98	0.79%