A HEURISTIC TO DETERMINE PRICE AND LOT SIZE IN FACE OF A SUPPLIER'S TEMPORARY PRICE REDUCTION OVER AN INTERVAL

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ABSTRACT

Assuming price sensitive demand, a recent paper presents a model for a reseller's response to a supplier's temporary price reduction over an interval. However, the paper proposes rather complicated computational procedures for obtaining the optimal solutions to the model. We show that effective use of Excel Solver can simplify the solution procedure for the practicing mangers. Thus, our work should help improve the likelihood of actual use of the model in practice. Secondly, we identify an inconsistency in the optimal strategies recommended by that model for the two situations of temporary price reductions it addresses. We show that the inconsistency originates in an unnecessarily restrictive assumption. Using a less restrictive assumption, we present a modified model. Because we do not even try to establish the existence of local optimality, our approach must be labeled as "a heuristic." However, our model and solution procedure not only produces internally consistent results, it also yields a higher profit for the reseller. In short, we present a model that is less restrictive, more profitable, and easier for practicing managers to actually use. Finally, based on the analysis in this paper, we suggest that instead of oversimplifying a situation just to be able to establish local or global optimality of the resulting solution, operations researchers should develop models that truly represent the particular situations they are modeling. Then, if necessary, they may want to use heuristic approaches to solve those models even if those approaches may not provide a guarantee for local or global optimality of the solution.

INTRODUCTION

How a reseller should respond to a supplier's temporary price reduction has been a subject of several studies. Most prior works assumed that the price reduction was offered only at one point in time (Abad, 1997; Arcelus & Srinivasan, 1998) and/or that the end demand was not sensitive to the reseller's selling price (Lev & Weiss, 1990; Arcelus & Srinivasan, 1991, Goyal, et al., 1991). In a recent paper, Abad (2003) makes a significant contribution by relaxing these restrictive assumptions.

Assuming price sensitive demand, Abad (2003) presents a model for a reseller's response to a supplier's temporary price reduction over an interval. He considers two types of temporary price reduction situations faced by a reseller:

- <u>Situation 1</u>. The discount is applicable only to units resold during the promotion. The reseller must resell all the units bought at discount within the promotion period and at a price lower than the reseller's normal price for the product.
- <u>Situation 2</u>. The discount is applicable on units purchased during the supplier's promotion. It is up to the reseller whether the discounted units are resold during the promotion period or later, and whether the quantity purchased on discount is resold at a reduced price or not.

In each one of the situations he addresses, assuming continuous variables, Abad (2003) establishes the first order conditions for optimality. However, since one of his decision variables must be integer valued, he then outlines a complicated procedure (requiring manual iterations) to arrive at the optimal integer value of that variable and the corresponding optimal values of the remaining variables. Real world managers may find that procedure to be too burdensome, if not intimidating. Given that Abad's (2003) recommended procedure involves the use of Solver, we outline a simpler procedure that should enhance the likelihood of use of his model in real life.

In his model addressing Situation 2, Abad (2003) assumes that, towards the end of the promotion period, the reseller would purchase a large lot at the discounted price, resell some part of that large lot at the same price used during the promotion period and then sell the rest of the lot at the reseller's regular price. We show that this assumption makes Abad's (2003) results for Situation 2 inconsistent with his results for Situation 1. We present a modified model for Situation 2 by relaxing Abad's restrictive assumption. Consistent with the exact conditions of the situation, we assume that the reseller has no restrictions whatsoever on the prices he can charge for the first or the second part of the large lot. In addition to producing results that are consistent with the results of Situation 1, our Situation 2 model provides price and order quantity strategies that are less restrictive and that result in a greater profit for the reseller.

In what follows, we first recapitulate Abad's (2003) model and his numerical example for all the situations he addresses. We show that when its full capabilities are used, Solver can yield practically the same optimal solutions that Abad (2003) obtains to his models of the various situations without resorting to the complicated computational procedure outlined by Abad (2003). Furthermore, Solver allows us to take care of such practical considerations as prices must be in integer pennies and order quantities must be integer numbers. We also identify an inconsistency in the optimal decisions recommended by Abad's (2003) model for the two situations it addresses.

Then, we modify Abad's (2003) model for Situation 2. Our model allows the reseller to charge any price he desires for any portion of the last order quantity bought on sale. Although our solution procedure is a heuristic, we show that this modified model results in strategies that are consistent with Abad's (2003) optimal strategies for Situation 1. We also show that our model results in a higher profit for a reseller facing Situation 2.

A RECAPITULATION OF ABAD'S (2003) MODEL

Abad (2003) assumes that demand is a known and stationary function of the reseller's selling price. When the supplier has offered no temporary discount, the situation is considered "a regular situation" Based on earlier works by Whitin (1955) and Kunreuther & Richard (1971), Abad (2003) assumes that, in the regular situation, the reseller would use a single order quantity for all the orders and a single resale price throughout the year. Abad (2003) develops the following model for the reseller's annual profit under the regular situation.

v = Regular per unit price charged by the supplier to the reseller
C = Fixed ordering cost for the reseller/ order
r = Reseller's holding cost per dollar per year
p = Reseller's selling price/ unit
$D(p) = D$ = The known and stationary demand function faced by the reseller.
$D(0) < \infty$, $D(p_u) = 0$ for some $p_u \in (0, \infty)$. $D' = \mathrm{d}D(p)/\mathrm{d}p < 0$ for $p \in (0, p_u)$.
Q = Reseller's order quantity

$W(p, Q)$ = Reseller's annual profit in the regular situation which is a function of the reseller's selling price and regular order quantity.

$$W(p, Q) = [p - v - (C/Q)]D(p) - Qrv/2. \quad (1)$$

In his numerical example, Abad (2003) uses:
v = \$8/unit, C = \$80/order, r = \$0.50/\$/yr, and $D(p) = 10000000p^{-3}$

Although Abad (2003) provides equations for the closed-form solution to the maximization of Equation (1), we do not need to recapitulate those. In this paper, we simply use Excel Solver (hereafter referred to as Solver) to obtain the optimal solutions to various problems. Solver can maximize smooth functions, such as the one in Equation (1), with no difficulty whatsoever. Hence, we need not worry about solving Abad's (2003) closed-form equations. Indeed, Abad (2003) himself reports that he used Solver to obtain the numerical results for all the models in his paper.

The use of Solver also allows us to take care of such practical considerations as prices must be in integer pennies and order quantities must be integer valued. Of course, when these types of integer constraints are involved, the function becomes non-smooth. Solver can only guarantee local optimality when non-smooth functions are involved. However, we can obtain a very good local optimum by properly setting Solver "options" on the maximum time, maximum iterations, precision level, and tolerance.

Using our Solver procedure and Abad's (2003) values of the parameters, the reseller's optimal order quantity is 466 units/order and his optimal selling price is \$12.26/unit. At this selling price, the reseller's annual demand rate is 5,427 units/yr and his profit is \$21,253.75/year. The numbers we have obtained are practically identical to those reported by Abad (2003). However, Abad (2003) reports the reseller's demand at the optimal price of \$12.26/unit to be 5,423 units/year and his profit to be \$21,254/year. Initially, we thought that the differences between the numbers reported by Abad (2003) and those obtained by us might be attributable to our use of the integer constraints on prices in pennies and order quantities. However, when those constraints are relaxed, the optimal price works out to be \$12.2575/unit, the corresponding demand works out to be 5,430 units/year, and the reseller's profit works out to be \$21,253.75/year. Thus, Abad (2003) seems to have simply rounded the calculated profit. These results are summarized in Table 1.

It should also be clear from the above similarities and differences in the numerical results obtained by us and those reported by Abad (2003), rounding of the numerical values for optimal price and order quantity does affect the calculated demand but not the profit for the reseller. Thus, for the modeled solution, we need not insist on prices being in integer pennies or order quantities being in integers, even though in real life they will have to be.

Situation 1

Next, Abad (2003) considers a situation where the supplier has access to the reseller's point-of-sale data and therefore can monitor the selling price and the quantity sold by the reseller under a particular selling price. The assumption is that the reseller qualifies for a discount only on units bought and resold during the promotion period and only if the resale price of those units is smaller than the reseller's regular price. We shall refer to this situation as Situation 1 of supplier's promotion. Based on earlier research in inventory theory, Abad (2003) notes that in this situation, the reseller's optimal strategy is to order and resell one or more integer valued equal size lots during the promotion period. To the notation in the regular situation, Abad (2003) now adds:

p_0 = Reseller's optimal price in the regular situation.
Q_0 = Reseller's optimal order quantity in the regular situation.
W_0 = Reseller's optimal annual profit in the regular situation.
d = The temporary price reduction (\$/unit) offered by the supplier to the reseller
$0 \leq d \leq v$.
T = The duration (in years) over which the supplier has offered a promotion or temporary price reduction.
$T \geq 0$.
m = The number of equal size lots purchased by the reseller during the supplier's temporary promotion.
$m \geq 0$ and integer.
If Q is the quantity ordered in each of the equal size lots, and p is the resale price for that quantity, in Situation 1,

$$Q = D(p)T/m = DT/m. \tag{2}$$

$\pi(p, m)$ = Reseller's incremental profit (i.e., revenue – costs of goods – carrying cost – ordering cost – regular profit) over the promotion period T of Situation 1.

$$\pi(p, m) = (p - v + d)DT - r(v - d)\, D(p)T^2/(2m) - mC - T\, W_0. \tag{3}$$

Thus, in this situation, the reseller has only two decision variables, p and m. As Equation (2) shows, the optimal Q will simply follow from these decisions. Abad (2003) derives the first order conditions for the maximization of Equation (3) above. He proves that at any given value of m, the profit function is unimodal. Given that unimodality, when the simultaneous solution of his first order conditions results in a fractional value for the number of lots purchased, Abad (2003) recommends a manual iterative procedure to determine the optimal integer value for the number of lots. His recommendation is to follow up the initial solution by using Solver twice, once when the number of lots is fixed at an integer value just below the calculated optimal value, and again when it is fixed at an integer value just above the calculated optimal value. The integer value that results in the higher value of Equation (3) is declared as the optimal number of lots.

As we see it, we do not need to recapitulate Abad's (2003) first order conditions, nor do we need to worry about obtaining fractional values for the number of lots. We simply use Solver to solve for the optimal price and optimal number of lots that maximize equation (3) with the constraint that the number of lots must be integer. Our Solver constraints also specify that all prices be in integer pennies (although as we pointed out before, this condition makes no material difference to the reseller's profit) and that the resale price be greater than the reseller's cost per unit but less than the reseller's regular price. In Situation 1, we no longer require integer values for the order quantities since order quantity is not a decision variable.

To ensure that Solver obtains a very good local optimum efficiently enough, we modify Solver's default "options" to allow a maximum time of 1,000 seconds, a maximum of 10,000 iterations, a precision level of 0.000000001, and a tolerance of 0.01%. With our approach, we find that there is no need for any manual iterations. As we have pointed out before, this or any other Solver-based procedure (including Abad's) will not really guarantee a globally optimal solution when there are integer constraints on certain variables. Frontline Systems, the producer of Excel Solver markets a "Premium Solver Platform," that can (automatically using multiple starting points for the decision variables of a model) maximize the chances of obtaining the globally optimal solution to a problem involving integer constraints. However, having implemented Abad's (2003) procedure as well as ours (using standard Excel Solver), we can report that, in practical terms, the solution we obtain is identical to the solution Abad's (2003) procedure obtains. Furthermore, compared to Abad's (2003) manual procedure, our simple procedure should make it more likely that Abad's (2003) model is actually used by practicing managers.

Abad (2003) does not provide a numerical example for Situation 1. However, in Situation 2 of temporary price reduction, Abad (2003) assumes that the supplier has offered a discount of $0.80/unit over a promotion period of 0.25 year. Using those parameter values in Situation 1 and employing our Solver procedure, the reseller's optimal policy works out as: Order 621 units/order, three times during the promotion period, and resell that order quantity at $11.03/unit. For the reseller, this policy results in an incremental profit of $1,302.41 during the promotion period. (See the middle portion of Table 1).

Situation 2

Next, Abad (2003) considers what we have labeled as Situation 2 of supplier's promotion. As in Situation 1, in Situation 2 also, the supplier again offers a temporary price reduction over a time interval. However, here the supplier cannot (or does not want to) monitor the reseller's pricing policies and resale timing. As in Situation 1, here also, the reseller's optimal strategy consists of ordering and reselling one or more integer valued equal size lots during the promotion period. In addition, in situation 2, just before the end of the promotion period, the reseller can purchase a large quantity at the discounted price and resell any portion of that quantity at any price the reseller desires at any time after the promotion period.

Unfortunately, Abad (2003) puts an unnecessary constraint on the resale of the large quantity purchased just before the end of Situation 2 promotion. He assumes that the reseller will sell a portion of the large quantity at the exact resale price used in Situation 1 and then sell the remaining portion at only the optimal regular price. To the notation and equations of the regular situation and Situation 1, Abad (2003) now adds the following:

θ = Duration of the first portion the resale of the large lot purchased just before the end of Situation 2 promotion. By Abad's assumption, during this time the reseller maintains the same reduced selling price, p, used during the actual promotion period, T, which required integer number of equal size lots. $\theta \geq 0$.

ψ = Duration of the second portion of the resale of the large lot purchased just before the end of Situation 2 promotion. By Abad's assumption, during this period, the reseller uses the optimal price in the regular situation, p_0. $\psi \geq 0$.

$\pi(p, m, \theta, \psi)$ = Reseller's incremental profit (i.e., revenue – costs of goods – carrying cost – ordering cost – regular profit) from all the inventory purchases made during Situation 2 promotion.

$$\pi(p, m, \theta, \psi) = (p - v + d)D(T + \theta) + (p_0 - v + d)D_0\psi - (m + 1)C - r(v - d)(DT^2/(2m) + D\theta^2/2 + D\psi^2/2 + \psi D_0\theta) - (T + \theta + \psi)W_0. \quad (4)$$

Clearly, this situation has four decision variables: the discounted resale price, the number of equal size lots, and the durations of the first and the second portion of the large lot inventory cycle. Abad (2003) derives the first order conditions for the maximization of Equation (4) and puts forward several propositions. Based on those propositions, he outlines a complicated manual iterative procedure involving four steps that would converge to locally optimal solution to the problem. Abad's (2003) procedure does require the use of Excel Solver at various stages. Also, he warns that his procedure can guarantee only a locally optimal solution. For increasing one's chances of obtaining a globally optimal solution, he recommends trying several different starting values of *p*. We believe that practicing managers would find Abad's (2003) procedure too burdensome, if not intimidating. A simpler recommendation would be to use the Premium Solver Platform. However, we find that for his model, neither Abad's (2003) procedure nor the Premium Solver Platform is really necessary if one uses full capabilities of Solver.

We simply use Solver to obtain the optimal solution by maximizing Equation (4) with respect to the four decision variables with appropriate constraints on the acceptable values of the decision variables. As before, to maximize the chance that Solver obtains a very good local optimum efficiently enough, we modify Solver's "options" suitably. Of course, we cannot be sure of obtaining the globally optimal solution, just as Abad (2003) cannot be. However, as the results below (obtained by our procedure) show, our simple procedure works.

Using Abad's (2003) parameter values, namely, a discount of $0.80/unit and the promotion period of 0.25 year, the final solution works out to be a resale price of $11.10/unit during the 3 identical inventory cycles and the 0.156 year first portion of the large lot cycle, followed by 0.162 year of the second part of the large lot inventory cycle when the resale price is same as the regular optimal price of $12.26/unit. This means that three times during the sales promotion period, the reseller should order 609 units/order and sell them at $11.10/unit. Just before the end of the promotion period, he should order 2,017 units, sell 1,139 of those units at $11.10/unit over a period of 0.156 year, and sell the remaining 878 units at the regular price of $12.26/unit over a period of 0.162 year.

This solution is the same as the one obtained by Abad (2003) for this situation. The only thing that is different is that while Abad (2003) reports an incremental profit of $2,289.6 for this situation, our calculations show a profit of only $2,289.30. Initially, we thought that this difference was due to the additional practical constraint we had imposed, namely that selling prices must be integer valued in pennies. However, a careful examination showed that even when we dropped the requirement for the prices to be in integer pennies, the prices changed fractionally and the reseller's profit worked out to be $2,289.37. Thus, we believe that Abad (2003) simply reported an incorrect number for the profit.

So far, we have recapitulated Abad's (2003) model for the two situations of temporary discount he has envisioned. We have shown that, if one uses Solver's full capabilities, one does not really need the complicated computational procedures proposed by Abad (2003) to solve the optimization problems of these situations. The simpler computational procedures we have outlined should make the actual use of Abad's (2003) model by real life resellers more likely.

We find it curious that, even though the supplier's discount amount and the promotion period duration are identical for the numerical examples for both Situation 1 and 2, the optimal price and order quantity decisions provided by Abad's (2003) model for the identical inventory cycles in those two situations are different. For example, in Situation 1, the optimal order quantity for the identical inventory cycles is 621 units/order and the optimal resale price is \$11.03/unit. Whereas in Situation 2, the optimal order quantity for the identical inventory cycles is 609 units/order and the optimal resale price is \$11.10/unit. (See Table 1).

We believe the root of this inconsistency lies in the fact that, in dealing with the Situation 2, Abad (2003) has made an unnecessarily restrictive assumption, namely that the identical inventory cycles will be followed by a large order quantity, a portion of which is sold at the same price used during the identical inventory cycles and the remaining quantity is sold at the reseller's regular price. In the next section, we modify Abad's (2003) model to relax this assumption.

A MODIFIED MODEL FOR RESELLER'S RESPONSE IN SITUATION 2

As we have pointed out before, in Situation 2 of a supplier's promotion, the reseller is free to set any selling price for any portion of the quantity bought on discount. As in Abad's (2003) model, we assume that there will be one or more integer valued identical cycles during the promotion period in which the reseller orders a quantity and sells it at a reduced resale price. At the end of those identical cycles and just before the promotion period expires, the reseller orders a large quantity. We also assume that the retailer will resell some portion of that large order quantity at one price and the remaining portion at another price. However, unlike Abad (2003), we do not assume that the first portion must be resold at the same price as the resale price used in the identical inventory cycles, nor do we assume that the second portion must be sold at the reseller's optimal regular price. As we see it, since the supplier cannot (or does not want to) monitor the timing or the price of the resale, the reseller is free to choose any price for any portion of his large order quantity.

To develop our modified model, let us add the following to the notation already defined.

Q_1 = Order quantity for the equal size lots during the promotion period.
p_1 = The resale price for each one of the units in Q_1.
Q_2+Q_3 = The total size of the large lot ordered just before end of the promotion period, to be sold over the duration $\theta+\psi$.
Q_2 = The quantity to be resold during time span $[T, T+\theta]$ at price p_2.
Q_3 = The quantity to be resold during time span $[T+\theta, T+\theta+\psi]$ at price p_3.
If D_1, D_2, and D_3, represent the reseller's annual demand rate at the prices p_1, p_2, and p_3, respectively, the following conditions hold:

$$Q_1 = D_1T/m. \qquad Q_2 = D_2\theta. \qquad Q_3 = D_3\psi. \tag{5}$$

$\pi(p_1, p_2, p_3, m, \theta, \psi)$ = Reseller's incremental profit from all the inventory purchased during Situation 2 promotion

$$\pi(p_1, p_2, p_3, m, \theta, \psi) = (p_1 - v + d)D_1T + (p_2 - v + d)D_2\theta + (p_3 - v + d)D_3\psi - (m+1)C - r(v-d)((D_1T^2/(2m)) + (D_2\theta^2/2) + (D_3\psi^2/2) + D_3\theta\psi) - (T+\theta+\psi)W_0. \tag{6}$$

Note that in our model, there are six decision variables. A mathematically elegant solution to this model would be far more complex than that for Abad's (2003) model involving only four decision variables. Even establishing the conditions for the existence of local optimality would require considerable effort and many pages of the paper. Perhaps this is precisely why Abad (2003) used the restrictive assumptions he used. However, given an input of appropriate constraints on the acceptable values of these decision variables, Solver can obtain an optimal solution to the problem. There would be no guarantee of a globally optimal solution, but to ensure that Solver has the greatest chance of obtaining a very good local optimum efficiently enough, we modify Solver's "options" to allow a maximum time of 1,000 seconds, a maximum of 10,000 iterations, and seek a precision level of 0.000000001. This modification is particularly important for our Situation 2 model since it has six decision variables. Now, for the same numerical values of the parameters used by Abad (2003), we obtain the following solution.

We obtain 3 identical inventory cycles with an order quantity of 621 units/order to be resold at $11.03/unit. Note that this is exactly the solution to Situation 1 problem. Thus, with our model, the solutions to the two types of temporary sale situations are mutually consistent.

For the large order just before the end of the temporary discount period, we obtain a total order quantity of 2005 units. Of those, 1,060 units should be sold at $11.20/unit over the first 0.149 year period after the promotion ends and 945 units should be sold at $12.05/unit over another 0.165 year period. The reseller's optimal incremental profit from the entire inventory purchased during the Situation 2 promotion is $2,294.25. Notice that this profit is higher, albeit modestly, than the profit ($2,289.37) produced by Abad's (2003) model. The last portion of Table 1 provides this comparison in a summary form.

Because we have not even tried to establish the existence of local optimality, our approach must be labeled as "a heuristic." However, as far as we are concerned, our approach does produce a feasible solution that meets every requirement of the situation and that produces a better profit than Abad's (2003) model does.

CONCLUSION

Assuming price sensitive demand, Abad (2003) presented a model for a reseller's response to a supplier's temporary price reduction over an interval. Even though he used Solver, Abad (2003) proposed rather complicated computational procedures to arrive at the optimal solutions to his model. Abad's (2003) computational procedures may be too intimidating for many managers. As such, they may be reluctant to use his model. We have shown that if one uses Solver's full capabilities, one does not need such complex computational procedures. The simpler computational procedures outlined here should make it more likely that Abad's (2003) model will be actually used by practicing managers.

Second, there was an inconsistency in the optimal strategies recommended by Abad's (2003) model for the two situations of temporary price reductions he addressed. We have shown that the inconsistency results from an unnecessarily restrictive assumption. Using a less restrictive assumption, we presented a modified model and an Excel Solver based heuristic to solve it. Our model not only produced internally consistent results, it also yielded a higher profit for the reseller. Although that increase in the profit is modest under the parameters values considered, it could be significant in situations of highly elastic demand and sizable inventory ordering and carrying costs. What is more important is that we now have a less restrictive model for a reseller who wants to respond to temporary price reductions from his supplier. Thus, our modification has added some value to Abad's (2003) model.

Finally, based on this analysis we want to observe an important facet of some of the current modeling efforts in operations research. As Abad (2003) has done, many OR authors impose simplifying restrictions on a modeling situation simply to keep the number of decision variables manageable enough to establish the existence of local or global optimality. In the process, sometimes they distort the true properties of the situation, and consequently, obtain a suboptimal solution. We believe it is better to model the situation true to its actual conditions even if one must resort to a heuristic solution procedure that produces a better solution than the one produced by the simplified model, knowing fully well that the heuristic approach may not guarantee local or global optimality.

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Table 1
A Comparison of Our Approach with Abad's Approach

Base case - Regular Situation (i.e., No Promotion)

	Abad's Procedure	Our Procedure
Common Assumptions	v = $8/unit, C = $80/order, r = $0.50/$/yr, and $D(p)$ = 10000000p^{-3}	
Unique Assumptions	Retail prices can be in fractional pennies Order quantities can be fractional	Retail prices must be in integer pennies Order quantities must be integer
Optimal Decisions	p as reported by Abad = $12.26/unit p as calculated by Excel = $12.2575	p_0 = $12.26/unit
	Q as reported by Abad = 466 units/order Q as calculated by Excel = 466.05 units/order	Q_0 = 466/order
Consequences	D as reported by Abad = 5,423 units/year D as calculated by Excel = 5,429.96	D_0 = 5,426.61 units/year
	Profit as reported by Abad = $21,254 /year Profit as calculated by Excel = $21,253.75/year	Profit = $21,253.75/year

Situation 1 - Discount Applicable to Quantity Resold within Promotion Period

	Abad's Procedure	Our Procedure
Common Assumptions	v = $8/unit, C = $80/order, r = $0.50/$/yr, $D(p)$ = 10000000p^{-3}, d = 0.80/unit T =0.25 year	
Unique Assumptions	Retail prices can be in fractional pennies	Retail prices must be in integer pennies
Optimal Decisions	Abad provides no numerical example for this situation. However, the procedure he has described would yield the same results as ours.	p = $11.03/unit m = 3 cycles
Consequences		Q = 621 units/order Incremental Profit = $1,302.41

Situation 2 - Discount Applicable to Quantity Purchased During Promotion Period

	Abad's Model	Our Modified Model
Common Assumptions	v = $8/unit, C = $80/order, r = $0.50/$/yr, $D(p)$ = 10000000p^{-3}, d = 0.80/unit T =0.25 year	
Unique Assumptions	First portion of the large lot sold at the same price as earlier identical lots. The second portion of the large lot sold at the regular price. Retail prices can be in fractional pennies.	Either portion of the large lot can be resold at whatever prices the reseller chooses. Retail prices must be in integer pennies.
Optimal Decisions	p_1 = $11.10/unit m = 3 cycles $\theta = 0.156$ year $\psi = 0.162$ year	p_1 = $11.03/unit m = 3 cycles $\theta = 0.149$ year $\psi = 0.165$ year p_2 = $11.20/unit p_3 = $12.05/unit
Consequences	Incremental Profit as reported by Abad = $2,289.6 Incremental Profit as calculated by Excel = $2,289.37	Incremental Profit = $2,294.25