Title: CONTINUOUSLY INCREASING PRICE IN AN INVENTORY CYCLE: AN OPTIMAL STRATEGY FOR E-TAILERS

Topic Area: Decision Sciences

Authors: Prafulla Joglekar
Patrick Lee

Addresses: Prafulla Joglekar, Management Department, LaSalle University
1900 W. Olney Ave., Philadelphia, PA 19141-1199. USA
Email: Joglekar@lasalle.edu
Phone: (215) 951 1036
Fax: (215) 951 1271

Patrick Lee, IS/OM Department, Fairfield University
1073 North Benson Road, Fairfield, CT 06824
Email: plee@mail.fairfield.edu
Phone: (203) 254 4000 x2846
Fax: (203) 254 4105

Published in:
Proceedings of Hawaii International Conference on Business, Honolulu, Hawaii,
May 26 - 29, 2005. ISSN#: 1539-722X. pp. 1584-1598.

CONTINUOUSLY INCREASING PRICE IN AN INVENTORY CYCLE: AN OPTIMAL STRATEGY FOR E-TAILERS

ABSTRACT

Operations researchers have always assumed that when a product's unit cost is constant and its demand curve is known and stationary, a retailer of the product would find it optimal to buy a fixed quantity every time he buys and to sell the product at a single price throughout the year. However, during any inventory cycle, the retailer's inventory on hand and the inventory carrying costs are continuously declining functions of time. Thus, it seems appropriate that the retailer's optimal price strategy should be a continuously increasing function of time. Of course, in the past, implementation of such a strategy would have been impractical, if not impossible. However, in today's e-commerce environment, e-tailers can easily implement a continuously increasing price strategy in an inventory cycle.

We present a model that shows that, when the demand curve is known and stationary, an e-tailer is better off using a continuously increasing price strategy than using a constant price strategy within any inventory cycle. Sensitivity analysis shows that this strategy is particularly profitable when demand is highly price sensitive and the inventory ordering and carrying costs are high. Thus, our model may be particularly useful for e-tailers of undifferentiated commodities (high price elasticity), e-tailers of imported products (high ordering costs), e-tailers who also manufacture the product (high set up costs), e-tailers of perishable products (high carrying costs), and e-tailers of products subject to obsolescence (high carrying costs).

CONTINUOUSLY INCREASING PRICE IN AN INVENTORY CYCLE: AN OPTIMAL STRATEGY FOR E-TAILERS

INTRODUCTION

When a reseller of a product buys the product at a constant unit cost, incurs a fixed cost per order, stores the product at a constant carrying cost per unit of inventory per year, and faces a constant demand rate over an infinite horizon, the economic order quantity (EOQ) model tells us that the reseller's optimal strategy is to buy a fixed quantity every time he or she buys. Ignoring inventory related costs, classical price theory tells us that when a product's demand is price sensitive but the demand curve is known and stationary, the reseller's optimal strategy is to charge a single price throughout the year. Whitin (1955) was the first one to extend the inventory theory with the concepts from the price theory to investigate the simultaneous determination of price and order quantity decisions of a reseller. Although he never stated so explicitly, Whitin (1955) assumed that when all assumptions of the EOQ model are valid except that demand is price sensitive, with a known and stationary demand curve, a reseller's optimal strategy would be, once again, to buy a fixed quantity every time he buys and to sell it at a single price throughout the inventory cycle.

Kunreuther and Richard (1971) sought to show that, when demand is price elastic, centralized decision-making (using simultaneous determination of optimal price and order quantity) was superior to the common practice of decentralized decision-making whereby the price decisions of a reseller were made by the marketing department and given that price, the order quantity decisions were made by the operations department. Although Kunreuther and Richard (1971) were perhaps unaware of Whitin's (1955) paper, their model was very similar to Whitin's (1955) model. Assuming a known and stationary demand curve along with the remaining conditions of the EOQ model, they asserted: "Given constant marginal costs of holding and purchasing the goods, the firm will want to maintain the same price throughout the year" (Kunreuther and Richard, 1971 p. 173). What they did not realize is that, even though marginal holding costs are constant per unit, a firm's holding costs at any particular time within an inventory cycle are a function of inventory on hand, which itself is a function of the time from the beginning of the inventory cycle. As our research shows, in this situation, a constant price throughout the year is not optimal.

Over the five decades since Whitin's (1955) work, numerous authors (Tersine and Grasso, 1981; Arcelus and Srinivasan, 1987; Ardalan, 1991; Hall, 1992; Martin, 1994; Arcelus and Srinivasan, 1998; and Abad, 2003) have used Whitin's (1955) and Kunreuther and Richard's (1971) models as foundations to their own models. However, none of these authors have questioned Whitin's (1955) and Kunreuther and Richard's (1971) assumption that the reseller's optimal strategy would be to sell the product at a single price throughout the inventory cycle. Considering a situation of price sensitive demand, Abad (1997 and 2003) found that, in the case of a temporary sale with a forward buying opportunity, a reseller's optimal strategy is to charge two different prices during the last inventory cycle of the quantity bought on sale – a low price at the beginning of the inventory cycle and a higher price starting somewhere in the middle of the cycle. Yet, Abad (1997 and 2003) did not consider a similar strategy in the regular inventory cycle of a product with price sensitive demand.

In this paper we show that, when demand is price-sensitive, Whitin's (1955) and Kunreuther and Richard's (1971) assumption of a single price throughout an inventory cycle leads to suboptimal profits for the retailer. During an inventory cycle, a reseller's inventory level and carrying costs are a declining function of time. When a reseller faces a price-insensitive demand, his selling price is set arbitrarily, since any optimizing model would push the price to infinity. In other words, in that situation, price is not seen as a decision variable for any mathematical model. Given an arbitrary price (and corresponding demand), the reseller's only strategy is to minimize his inventory ordering and holding costs by using the EOQ model. However, in today's e-commerce environment, when demand is price sensitive, an e-tailer (e.g., Amazon.com) can adopt a continuously increasing price strategy to minimize the impact of the time-dependent inventory carrying costs. The idea is to charge a relatively low selling price at the beginning of an inventory cycle when the on-hand inventory is large. The low price would generate high demand. Consequently, the inventory level as well as the marginal inventory carrying costs would decline rapidly. However, as the inventory cycle progresses, and as the on-hand inventory and the marginal carrying costs decline, the e-tailer can charge increasingly higher prices to maximize profit.

With the widespread use of revenue management or yield management techniques (Feng and Xiao, 2000; McGill and Van Ryzin, 1999; Smith, et al. 1992; Talluri and van Ryzin, 2002; Weatherford and Bodily, 2000) in the airline, car rental, and hotel industries today, a time-dependent pricing strategy has become commonly adopted. Although revenue management techniques are typically applied in situations characterized by product perishability, fixed capacity and a possibility of market segmentation (Talluri and van Ryzin, 2002), we show that, even in the absence of these characteristics, in an e-commerce environment, a dynamic pricing strategy makes sense. We present a model that shows that, when the demand curve is known and stationary, an e-tailer is better off using a continuously increasing price strategy.

In what follows, first we recapitulate Whitin's (1955) and Kunreuther and Richard's (1971) fixed price strategy model. In the next section, we present our own model. Then, we provide several numerical examples considering a linear demand curve and varied values of relevant parameters. The final section provides the conclusions of our analysis along with some directions for future research.

THE FIXED PRICE MODEL

Both Whitin (1955) and Kunreuther and Richard (1971) consider a situation where all the other assumptions of the EOQ model are valid but demand is price sensitive, with a known and stationary demand curve. Whitin's (1955) notation is different from Kunreuther and Richard's (1971) notation. There are also some minor differences in the details of the two models. However, the following captures the basics of both the models. Although the model is applicable to any form of the demand function, for simplicity, we use a linear demand function.

Let,
C = Retailer's known and constant unit cost of buying the product
S = Retailer's known and constant ordering cost per order
I = Retailer's carrying costs per dollar of inventory per year
P = Retailer's selling price per unit
    It is assumed that $P > C$.
D = Retailer's annual demand as a function of the selling price, P.
    $D = a - bP$, where a and b are non-negative constants, a representing the theoretical maximum annual demand (at the hypothetical price of \$0 per unit) and b representing the demand elasticity (i.e., the reduction in annual demand per dollar increase in price).
T = The duration of retailer's inventory cycle – a decision variable.
Q = Retailer's order quantity per order. $Q = DT$.
Z = Retailer's profit per year.

$$\begin{aligned} Z &= (P - C)D - IC(DT/2) - S/T \\ &= (P - C - ICT/2)(a - bP) - S/T \qquad (1) \end{aligned}$$

Differentiating Z with respect to P and T, the first order conditions for the maximization of this function are:

$$P^* = (½)(a/b + C) + ICT/4 \qquad (2)$$

And

$$T^* = \{2S/(IC(a - bP)\}^{1/2} \qquad (3)$$

We have verified that the second order conditions (not presented here) fulfill the requirements for a local maximum of the annual profit. Thus, by solving for Equations (2) and (3) simultaneously, one can obtain the optimal values of the price and the inventory cycle time. Unfortunately, to obtain a closed form solution to these equations, one must solve a cubic equation that has three possible roots for the optimal value of T. Of course, we would be interested in only the real root(s). There is no simple closed form approach to obtaining the real root(s) of a cubic equation. Hence, in our numerical examples, we shall rely on Excel Solver to obtain the optimal solution to this problem. Of course, when there are multiple real solutions to a cubic equation, Excel gives only one of those solutions. In theory, one should use a software such as MathCAD to obtain all of the possible solutions and then determine which one is the global optimal. We did carry out this strategy for several numerical examples. In all the cases, what we found was that only one (if any) of the three solutions results in a feasible, and hence optimal, solution. Therefore, in Excel Solver, we input feasibility conditions such as price must be greater than per unit cost, order quantity must be positive, and annual profit must be positive. Excel also needs reasonable starting values of the decision variables. Once care is taken to input these conditions and reasonable starting values, in our experimentation, Excel Solver has never failed to return the best real solution (if any). Hence, we believe that practicing managers would be adequately served by the use of Excel Solver. They need not worry about using MathCAD.

THE CONTINUOUSLY INCREASING PRICE MODEL

We retain all of the assumptions of the foregoing model except now we assume that the retailer uses a continuously increasing price strategy within each inventory cycle.

Let us add the following notation,

t = time elapsed from the beginning of an inventory cycle
$P(t) = f + gt$ where f and g are non-negative decision variables, and $f > C$.
f represents the retail price at the beginning of each inventory cycle and g represents the annual rate of increase in the retail price throughout an inventory cycle.

Given that price is a function of time, now the retailer's annual demand rate will also be a function of time. Hence, we should redefine demand as

$$D(t) = a - b\,P(t) = a - bf - bgt \tag{4}$$

Also let,
$X(t)$ = Retailer's inventory at time t

Since at the beginning of the inventory cycle, the retailer orders a quantity Q to meet the cycle time demand,

$$Q = X(0) = \int_0^T D(t)\mathrm{d}t = \int_0^T (a - bf - bgt)\mathrm{d}t = (a - bf)T - (bg/2)T^2 \tag{5}$$

And for a general value of t,

$$X(t) = \int_0^T D(t)\mathrm{d}t - \int_0^t D(t)\mathrm{d}t$$
$$= (a - bf)T - (bg/2)T^2 - (a - bf)t + (bg/2)t^2 \tag{6}$$

Let Y = Retailer's profit per inventory cycle

Y is given by

$$Y = \int_0^T [P(t) - C]D(t)\mathrm{d}t - IC\int_0^T X(t)\mathrm{d}t - S \tag{7}$$

When simplified, this yields

$$Y = (a - bf)(f - C - ICT/2)T + (a - 2bf + bC)gT^2/2 + bg(IC - g)T^3/3 - S \tag{8}$$

Thus, the retailer's annual profit is given by

$$Z = Y/T = (a - bf)(f - C - ICT/2) + (a - 2bf + bC)gT/2 + bg(IC - g)T^2/3 - S/T \qquad (9)$$

To derive the first order conditions, differentiating Z with respect to f and equating it to zero, we obtain

$$f^* = (½)(a/b + C) + (IC/4 - g^*/2)T \qquad (10)$$

Differentiating Z with respect to g and equating it to zero, we obtain

$$f^* = (½)(a/b + C) + (IC/3 - 2g^*/3)T \qquad (11)$$

From Equations (10) and (11), we can see that the optimal values of g and f are given by (12) and (13) below

$$g^* = IC/2 \qquad (12)$$

$$f^* = (½)(a/b + C) \qquad (13)$$

An interesting thing to note here is the optimal value of g^***,** the price increase per year. It is one half of the cost of carrying one unit for one year. In other words the retailer's optimal selling price at the beginning of the cycle will be f^* while his optimal selling price for a unit at the end of the cycle will be ($f^* + ICT/2$). This makes intuitive sense. A unit in stock at the beginning of an inventory cycle, if unsold then, will be carried (on an average) over one of the duration of the cycle. Hence it would cost the retailer $ICT/2$ to carry it. Whereas a unit in stock at the end of the cycle will not incur this cost. Hence, the retailer can afford to charge $ICT/2$ less (and save that carrying cost) for a unit at the beginning of the cycle compared to the price for a unit at the end of the cycle. In the process, the retailer benefits from the greater demand generated by that lower price in the early part of an inventory cycle.

Now, differentiating Z with respect to T, equating it to zero, and substituting the optimal values of f and g from (12) and (13), we obtain

$$T^3 + [3(bC - a)/(2bIC)]T^2 + 6S/(bI^2C^2) = 0 \qquad (14)$$

Given that (14) is a cubic equation, we have three possible roots for the optimal value of T. Of course, we would be interested in only the real root(s). As we said in the context of the fixed price model, there is no simple closed form approach to obtaining the real root(s) of a cubic equation. Hence, in our numerical examples, we rely on Excel Solver to obtain the optimal solution to this problem. As in the case of the fixed price model, we input several feasibility conditions and provide reasonable starting values for the decision variables. We are happy to report that in our experimentation with this model also, Excel Solver has never failed to return the best real solution (if it exists). Hence, we believe that practicing managers would be adequately served by the use of Excel Solver.

We tried to verify whether the second order conditions of this model (not presented here) fulfill the requirements for a local maximum of the annual profit. Unfortunately, the results are not conclusive. They indicate either a local maximum or an inflection point. Thus, once again, we will have to first obtain a numerical solution to *T* and then verify if that solution gives a local maximum for the retailer's profit by perturbing the solution value of *T* and checking its impact on the profit value. In our numerical experiments with this model, the solution given by Excel Solver was always a local optimum (if it existed) rather than an inflexion point.

Let us now turn to a numerical example.

NUMERICAL EXAMPLE – BASE CASE

Consider a situation where the retailer's cost of a product is $7 per unit. The theoretical maximum annual demand is 50,000 units and demand declines at the rate of 5,000 units for each dollar's increase in the price. The ordering costs are $400 per order and the carrying costs are $0.40 per dollar of inventory. We shall refer to this set of assumptions as the base case.

Table 1 summarizes the base case assumptions, optimal decisions and consequences under the two models. As can be seen there, in the fixed price model, the optimal retail price is $8.64 per unit and the optimal inventory cycle time is 0.2053 years. This means that the retailer would order 1392 units per order and would obtain a per cycle profit of $1,487.96. The retailer's annual demand is 6790 units and his profit under this strategy is $7,249.24 per year.

In the continuously increasing price strategy model, the optimal retail price is $8.50 per unit at the beginning of the cycle and that price increases at the rate of $1.40 per year. The optimal cycle time is 0.2093 years. This means that the retail price at the end of the cycle is $8.79 per unit, the retailer would order 1,416 units per order, and his per cycle profit under this strategy would be $1,524.47. The retailer's annual demand would be 6,775 units and his annual profit would be $7,284.32.

In addition to reporting these numbers, Table 1 also presents the percent difference between the two models for each relevant decision and consequence. Observe that, in percent terms, the differences are small. Compared to the fixed price strategy, the continuously increasing price strategy results in a slightly longer cycle time. At the beginning of the inventory cycle, under the continuously increasing price model, the retail price is smaller than what it is under the fixed price model. However, by the end of the cycle, the retail price under the continuously increasing price model is larger than what it is under the fixed price model. As a result, the annual demand is smaller under the continuously increasing price model. The per-cycle profit shows a 2.45% improvement in the continuously increasing price model compared to the per cycle profit under the fixed price model. However, the annual profit is only 0.48% greater under the continuously increasing price model. Although this increase in the annual profit is modest, it is clear that, for an e-tailer, the continuously increasing price strategy is more profitable than the fixed price strategy. Furthermore, in a competitive market, every modest increase in profit is desirable.

Now, a skeptic might wonder whether the magnitude of the profit difference between the two models is worth all the fuss. A skeptic may argue that the continuously increasing price strategy model may be costlier to implement because it more complex and because it would call for additional efforts in communicating the pricing strategy to the consumer. However, as we see it, in a computerized society, this model is no more difficult to program than the fixed price model, and for an e-tailer to quote a time-dependent price is no more costly than quoting a fixed price when a customer is shopping at the e-tailer's website.

The specific numerical results we have obtained are a function of the numerical assumptions we have made. Hence, in order to identify the circumstances under which the continuously increasing price strategy would be particularly desirable, we carried out a sensitivity analysis, as described in the following section.

SENSITIVITY ANALYSIS

Table 2 summarizes the results of an analysis where we increased the value of each one of our parameters, one at a time, by 10% while maintaining the values of the other parameters constant. In each case, Table 2 shows the consequences of these changes on the retailer's annual profits under the two models and the percentage increase in the annual profit that the retailer obtains by using the continuously increasing price strategy as against using the fixed price strategy. For comparison purposes, the first row of Table 2 repeats the profit results of the two models in the base case.

Other things remaining unchanged from the base case, when the retailer's unit cost for the product goes up by 10% (from \$7 per unit to \$7.70 per unit), a retailer who uses the fixed price strategy would obtain an annual profit of \$2,993.58 while a retailer who uses the continuously increasing price strategy would realize a profit of \$3,048.31. Thus, in this situation, the continuously increasing price strategy affords an increase of 1.83% over the profit provided by the fixed price strategy. When these results are compared with the base case results, we can conclude that, other things remaining the same, when a retailer's product cost increases, the continuously increasing price strategy becomes substantially more desirable to use.

Table 2 also indicates that, other things remaining unchanged, if the theoretical maximum demand increases by 10%, the advantage of the continuously increasing price strategy reduces substantially to only 0.16% compared to the profit a retailer can make using the fixed price strategy. On the other hand, when demand elasticity increases by 10%, the continuously increasing price strategy seems to be the most desirable compared to all the cases of sensitivity analysis we have presented. In this case, it provides an annual profit that is 2.16% higher than the profit provided by the fixed price strategy. Other things remaining unchanged, when the ordering cost increases, the continuously increasing price strategy becomes a little more desirable than in the base case. Now the advantage is 0.55%. In terms of the retailer's profits from the two strategies, the impact of a 10% increase in inventory carrying costs is identical to that of a 10% increase in the ordering costs.

Thus, other things remaining the same, high price elasticity favors the continuously increasing price strategy the most, followed by high unit cost and then by high ordering and carrying costs. Thus, the continuously increasing price strategy is particularly desirable for e-tailers of undifferentiated commodities (high price elasticity), and e-tailers whose suppliers have considerable pricing power (high product cost). It should also be attractive to e-tailers of imported products (high ordering costs), e-tailers who also manufacture the product (high set up costs), and e-tailers of perishable products (high carrying costs) and e-tailers of products subject to sudden obsolescence (high carrying costs).

Of course, our sensitivity analysis focused on changes in one parameter at a time. When several parameters are favorable to the continuously increasing price strategy, the gains offered by the continuously increasing price strategy need not remain modest.

CONCLUSION

Traditionally, operations researchers (Whitin, 1955; Kunreuther and Richard, 1971; and others) have assumed that when a product's demand curve is known and stationary, a retailer of the product would find it optimal to buy a fixed quantity every time he buys and to sell the product at a fixed price throughout the year. We find that this assumption was wrong. Instead, we have shown that, within each inventory cycle, a continuously increasing price strategy is more desirable than the fixed price strategy. Our model resulted in an optimal increase per year of one half of unit holding cost, which made intuitive sense.

A continuously increasing price strategy might have been deemed impractical in the past. However, today's e-tailers can easily adopt this strategy.

Our numerical example suggests that often the advantage of a continuously increasing price strategy is only modest. However, sensitivity analysis shows that this strategy is particularly desirable when demand is highly price sensitive or when an e-tailer's supplier commands great pricing power. E-tailers facing high inventory ordering and carrying costs will also find this strategy fairly attractive.

While the continuously increasing price strategy may not be practical for a brick-and-mortar retailer, such a retailer could use the dual price strategy model developed by Joglekar (2003). Contending that, during any inventory cycle, the reseller's inventory carrying costs are a declining function of time, Joglekar (2003) showed that a retailer who sets two different prices at two different points in an inventory cycle obtains a greater profit than a retailer using a single fixed price throughout the cycle. Our numerical analysis of Joglekar's (2003) model (not presented here) showed that a continuously increasing price strategy is superior to his model. However, Joglekar's (2003) model's gains are only slightly inferior to the model we have presented here. Hence, a brick-and-mortar retailer may be well served by that model.

There are several directions in which this research can be extended. First we have assumed that the e-tailer obtains the product from a vendor. The model can be easily extended to a situation where the e-tailer is also a manufacturer of the product. We have already cited numerous works that have continued to use the single price assumption. Clearly each one of those models needs to be revisited in light of our findings here. Finally, a number of recent models considering the coordination pricing and order quantity decisions across a supply chain (Weng and Wong, 1993; Weng 1995a; Weng 1995b; and Boyaci and Gallego, 2002) have assumed a fixed price for each participant of the supply chain. These models also need to be updated by considering a continuously increasing price strategy.

REFERENCES

Abad P. L. (1997). Optimal policy for a retailer when the supplier offers a temporary reduction in price. *Decision Sciences*, 28, 637–49.

Abad P. L. (2003). Optimal price and lot size when the supplier offers a temporary price reduction over an interval. *Computers and Operations Research*, 30, 63-74.

Arcelus, F.J. & G. Srinivasan (1987). Inventory policies under various optimizing criteria and variable markup rates. *Management Science*, 33, 756-762.

Arcelus F. J. & G. Srinivasan (1998). Ordering policies under one time only discount and price sensitive demand. *IIE Transactions*, 30, 1057–64.

Ardalan, A. (1991). Combined optimal price and optimal inventory replenishment policies when a sale results in increase in demand. *Computers and Operations Research*, 18, 721-730.

Boyaci, T. and G. Gallego (2002). Coordinating pricing and inventory replenishment policies for one wholesaler and one or more geographically dispersed retailers. *International Journal of Production Economics.* **77,** 95-111.

Feng, Y. & B. Xiao (2000). A continuous-time yield management model with multiple prices and reversible price changes. *Management Science*, 46(5), 644-657.

Hall R. (1992). Price changes and order quantities: impacts of discount rate and storage costs. *IIE Transactions*, 24, 104–10.

Joglekar, P. (2003). Optimal price and order quantity strategies for the reseller of a product with price sensitive demand. *Proceedings of the Academy of Information and Management Sciences*, 7(1), 13-19.

Kunreuther, H. & J.F. Richard (1971). Optimal pricing and inventory decisions for non-seasonal items. *Econometrica*, 39, 173-175.

Martin, G.E. (1994). Note on an EOQ model with a temporary sale price. *International Journal of Production Economics*, 37, 241-243.

McGill J.I & G.J. van Ryzin (1999). Revenue management: Research overview and prospects. *Transportation Science*, 33, 233-256.

Smith, B.C., J.F. Leimkuhler & R.M. Darrow (1992). Yield management at American Airlines. Interfaces. 27, 8-31.

Talluri, K.T & G.J. van Ryzin (2002). *The theory and practice of revenue management.* Kluwer Academic Publishers, Dordrecht, The Netherlands.

Tersine, R.J. & R.L. Price (1981). Temporary price discounts and EOQ. *Journal of Purchasing and Materials Management*, 17, 23-27.

Weatherford, L.R. & S.E. Bodily (1992). A taxonomy and research overview of perishable-asset revenue management: Yield management, overbooking and pricing. *Operations Research.* 40, 831-844.

Weng, Z.K. (1995a). Channel coordination and quantity discounts. *Management Science.* 41**,** 1509-1522.

Weng, Z.K. (1995b). Modeling quantity discounts under General price-sensitive demand functions: optimal policies and relationships. *European Journal of Operational Research* **86,** 300-314.

Weng, Z.K. and R.T. Wong. (1993). General models for the supplier's all unit quantity discount policy. *Naval research Logistics.* 40**,** 971-991.

Whitin, T.M. (1955). Inventory control and price theory. *Management Science.* 2, 61-68.

Table 1
A Numerical Example

Assumptions Common to Both Models

C = \$7/unit; a = 50,000 units/year; b = 5,000 units/year;

S = \$400/order; I = \$.40/dollar/year

Optimal Decisions and Consequences under the Two Models

		Fixed Price Model	Continuously Increasing Price Model	Percent difference between the two models
Optimal Decisions	Cycle time (T)	0.2053 years	0.2093 years	1.95%
	Price at the beginning of the cycle (f)	\$8.64/unit	\$8.50/unit	-1.62%
	Price increase rate per year (g)	None	\$1.40/unit	NA
Consequences	Order quantity (Q)	1,392 units/order	1,416 units/order	1.72%
	Price at the end of the cycle ($= f + gT$)	\$8.64/unit	\$8.79/unit	1.74%
	Annual Demand	6,790 units/year	6,775 units/year	-0.22%
	Profit per cycle [$Z(T)$]	\$1,487.96/cycle	\$1,524.47/cycle	2.45%
	Profit per year (Z)	\$7,249.24/year	\$7,284.32/year	0.48%

Table 2

Sensitivity Analysis

Changed assumption (s)	A Comparison of the annual profit under the two models		
	Fixed Price Model	Continuously Increasing Price Model	Percent difference between the two models
None (Base Case)	$7,249.24/year	$7,284.32/year	0.48%
C = $7.70/unit	$2,993.58/year	$3,048.31/year	1.83%
a = 55,000 units/year	$15,339.34/year	$15,364.48/year	0.16%
b = 5,500 units/year	$2,568.27/year	$2,623.70/year	2.16%
S = $440/order	$7,059.27/year	$7,098.11/year	0.55%
I = $0.44/dollar/year	$7,059.27/year	$7,098.11/year	0.55%