On September 23, 1999, NASA’s $125 million Mars Climate Orbiter approached the red planet under guidance from a team of flight controllers at the Jet Propulsion Laboratory. It had been in flight for over nine months, covering more than 415 million miles of empty space on its way to Mars. As the Orbiter reached its final destination, the flight controllers began to realize that something was wrong. They had planned for the probe to reach an orbit approximately 180 km off the surface of Mars – well beyond the planet’s thin atmosphere. But new calculations based on the current flight trajectory showed the Orbiter skimming within 60 km of the Martian surface. The consequences were catastrophic: the spacecraft was incinerated by the friction from an atmospheric entry that it was never supposed to make.

What caused this disaster? NASA engineers had calculated the force necessary to maneuver the probe in units of pounds (a non-metric unit), while the JPL team worked in Newtons (a metric unit), and the software that calculated how long the thrusters should be fired did not make the proper conversion. The result is that the Orbiter delivered insufficient thrust to overcome the gravity of the giant planet.

The Orbiter loss illustrates the need for consistent use of units. Most people, however, are most comfortable working in whatever units they grew up using. As a result, unit consistency may not be possible within or between teams around the world. Ideally, people should be comfortable with a variety of ways of converting units in order to allow for collaboration among individuals from a variety of backgrounds.

While most people are not controlling NASA space probes, unit conversion is something that happens every day, in all walks of life. Even such a simple problem as figuring out that two dozen eggs equals 24 eggs is, at its heart, a unit conversion problem. Whether you realize it or not, when you do this problem in your head, you’re figuring it out like this:

\[
2 \text{ dozen eggs} \times \frac{12 \text{ eggs}}{1 \text{ dozen eggs}} = 24 \text{ eggs}
\]

**Dimensional Analysis**

Generally, unit conversions are most easily solved using a process called dimensional analysis, also known as the factor-label method. (A notable exception is the conversion among temperature units.) Dimensional analysis uses four fundamental steps to make these conversions:

1. Define the starting information and the desired result
2. Identify unit conversion factors that can be used to get from beginning to end
3. Set up a mathematical equation to perform the unit conversion
4. Cancel units that appear on both the top and bottom of ratios.
Demonstration of Method: Number of individual eggs present in 2 dozen eggs

1. The first step in solving a unit conversion problem is to define what it is you are trying to find and to identify what starting information you are being given. This will allow you to construct a strategy for getting from the beginning to the end. In the earlier example about eggs we knew we want to find the number of individual eggs given that we were initially told that we had 2 dozen eggs.

\[ ? \text{ eggs} = 2 \text{ dozen eggs} \]

(final answer) (starting information)

2. The next step is to identify conversion factors that will get us from the one end of the equation to the other. A conversion factor is a statement of the equal relationship between two units. In the egg problem the statement that “1 dozen eggs = 12 eggs” is a conversion equation. Since the two sides of the conversion equation are equal each other, then the ratio of the two should just be equal to 1. This is true irrespective of which side goes on the top or on the bottom.

\[ \frac{12 \text{ eggs}}{1 \text{ dozen eggs}} = 1 \quad \text{or} \quad \frac{1 \text{ dozen eggs}}{12 \text{ eggs}} = 1 \]

3. The next step in dimensional analysis is to set up a mathematical problem that uses one or more conversion factors to get to the units you are interested in. Mathematically you can multiply the starting data by as many factors of 1 (as many conversion factors) as you need. In the egg problem, if you have 2 dozen eggs and want to know how many individual eggs you have, you would set up the problem like this:

\[ 2 \text{ dozen eggs} \times \left( \frac{12 \text{ eggs}}{1 \text{ dozen eggs}} \right) = ? \]

4. Units, just like numbers or variables in an equation, “cancel” when you divide a unit by itself (i.e. when the unit is found on both the top and the bottom of a ratio). So the final step in dimensional analysis is to work the math problem such that the units cancel along the way. In the egg example, you started with 2 “dozen eggs” on the top, so you arrange the conversion factor so that its “dozen eggs” units are on the bottom. This cancels the “dozen eggs” units from both places in the equation, leaving you only with “eggs” for your final answer.

\[ 2 \text{ dozen eggs} \times \left( \frac{12 \text{ eggs}}{1 \text{ dozen eggs}} \right) = 24 \text{ eggs} \]
Let’s apply these steps to a slightly more complex problem than counting eggs… How much money would it cost to fill a truck’s 23 gallon gas tank if gas cost $2.87 per gallon?

1) Define starting and ending points: $ = 23 gal

2) Identify conversion factor, in this case the price: gallon = $2.87

3) Multiply starting information by conversion factor: \[ 23 \text{ gal} \times \frac{\$2.87}{1 \text{ gal}} = \]

4) Cancel out units that appear both top and bottom: \[ 23 \text{ gal} \times \frac{\$2.87}{1 \text{ gal}} = \$66.01 \]

Now that you’ve filled your tank, it’s time to head off for your day trip to Mexico. As you cross the border from the US into Mexico, you notice that the speed limit sign reads 100. Wow! Can you step on the gas, or is there something else going on here? There are very few countries other than the United States where you will find speeds in miles per hour – almost everywhere else they would be in kilometers per hour. So, some converting is in order to know what the speed limit would be in a unit you’re more familiar with.

First, we need to define what “100 kilometers per hour” means mathematically. The “per” tells you that the number is a ratio: 100 kilometers distance per 1 hour of time. Other than that, you need to know the conversion factor between kilometers and miles, namely 1 mile = 1.61 km. Now the set-up is pretty simple.

\[ \frac{\text{miles}}{\text{hour}} = \frac{100 \text{ km}}{\text{hour}} \]

\[ \frac{\text{miles}}{\text{hour}} = \frac{100 \text{ km}}{1.61 \text{ km}} = 62.1 \text{ mph} \]

So far you’ve seen examples with only one conversion factor, but this method can be used for more complicated situations. When it’s time to leave for home from your day trip in Mexico, you realize you have just enough gas to make it back across the border into the US before you have to fill up. You notice that you could buy gas for 6.50 pesos per liter before you head home. At first glance that seems more expensive than the $2.87 per gallon at home, but is it really? You need to convert to be sure. Fortunately you came prepared, and looked up the currency exchange rate (1 peso = 8.95 cents) and volume conversion (1 gallon = 3.79 Liters) before you left.

This conversion is more complicated than the previous examples for two reasons. First, you do not have a single direct conversion factor for the monetary conversion (pesos to dollars), but what you do know is that 1 peso = 8.95 cents and that 100 cents = 1 dollar. Together, these two facts will let you convert the currency. The second twist is that you are not only changing the
money unit – you also need to convert the volume unit as well. These two conversions can be done in the same equation. The order does not matter, but both must be done.

\[
\frac{\$}{\text{gallon}} = \frac{6.50 \text{ pesos}}{1 \text{ L}}
\]

\[
\frac{\$}{\text{gallon}} = \frac{6.50 \text{ pesos}}{1 \text{ L}} \times \left( \frac{8.95 \text{ ¢}}{1 \text{ peso}} \right) \times \left( \frac{1 \text{.00}}{100 \text{ ¢}} \right) \times \left( \frac{3.79 \text{ L}}{1 \text{ gal}} \right) = \frac{\$ 2.20}{1 \text{ gal}}
\]

Notice that the “L” had to be placed above the division bar in the conversion factor in order to cancel out the “L” below the division bar in the original number. Also note that even though the “L” terms are separated by two other conversion factors for the money, they still cancel each other out. Now it is easier to decide whether you should fill up before or after you return to the United States.

You can see that you don’t have to be an engineer at NASA to need dimensional analysis. You need to convert units in your everyday life (to budget for gas price increases, for example) as well as in scientific applications, like stoichiometry in chemistry and converting from English to metric units in physics. If you know what units you have to work with, and in what units you want your answer to be, you don’t need to memorize a formula. If the teams working on the Mars Climate Orbiter had realized that they needed to go through these steps, we would be getting weather forecasts for Mars today.
PROBLEM SET

For these problems, set up the answer but don't bother to do all the multiplications and divisions. The answers follow directly. After you set up the answer, immediately check to see if you have done it correctly. What you want to know is whether you have mastered the method of dimensional analysis.

1. How many seconds in 800 minutes?
2. How many minutes in 5 years?
3. How many years in 500 days?
4. How many dozens of eggs are there in 2500 eggs?
5. How many miles in 12,000 yards? There are 5280 ft/mi and 3 ft/yard.
6. How many decades are there in 9 centuries? There are 10 years in a decade.
7. What is the cost of 8 onions if 3 onions weigh 1.5 lb and the price of onions is 20¢ per lb?
8. How many hours does it take to drive to Los Angeles from San Francisco at an average speed of 52 mi/hr? The distance between the two cities is 405 mi.
9. What is the cost of driving from San Francisco to Los Angeles (405 mi) if the cost of gasoline is $1.34/gal and the car gets 15 mi/gal of gasoline?
10. How many oranges are in a crate if the price of a crate is $1.60 and the price of oranges is $0.30/lb? On the average there are three oranges per pound.
11. The price of a ream of paper is $4.00. There are 500 sheets of paper in a ream. A sheet of paper weighs 0.600 oz. What is the price per pound of paper? (There are 16 oz in a pound.)
12. How many cars are there in a long freight train if it takes the entire train 2 min to pass a station as it travels 40 mi/hr? Each car is 50 ft long. There are 5280 ft in a mile.
13. A crate of eggs holds 27 cartons of eggs. Each carton contains a dozen eggs. It is found that the eggs in the crate weigh 10 lb. What is the average weight of an egg in ounces? (There are 16 ounces per pound.)
PROBLEM SET ANSWERS

You should be able to do at least ten of these problems correctly. This chapter is so important that if you missed three or more problems, you had better go over the entire chapter again.

1. \[ \frac{\text{sec}}{\text{min}} = \frac{800 \text{ min} \times 60 \text{ sec}}{\text{min} \times 60 \text{ sec}} = 200 \times 60 \text{ sec} \]

2. \[ \frac{\text{min}}{\text{yr}} = \frac{5 \text{ yr} \times \frac{365 \text{ days}}{\text{yr}} \times 24 \text{ hr} \times 60 \text{ min}}{\text{hr} \times 3600 \text{ min}} = 5 \times 365 \times 24 \times 60 \text{ min} \]

3. \[ \frac{\text{yr}}{\text{days}} = \frac{500 \text{ days}}{365 \text{ days}} = 500 \times \frac{1}{365} \text{ yr} \]

4. \[ \frac{\text{doz}}{\text{eggs}} = \frac{3500 \text{ eggs}}{12 \text{ eggs}} = 3500 \times \frac{1}{12} \text{ doz} \]

5. \[ \frac{\text{mi}}{\text{yd}} = \frac{12,000 \text{ yd}}{\text{mi}} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{\frac{\text{mi}}{5280 \text{ ft}}}{\text{mi}} = 12,000 \times 3 \times \frac{1}{5280} \text{ mi} \]

6. \[ \frac{\text{decades}}{\text{centuries}} = \frac{9 \text{ centuries}}{100 \text{ yr}} \times \frac{100 \text{ yr}}{\text{century}} \times \frac{\text{decade}}{10 \text{ yr}} = 9 \times 100 \times \frac{1}{10} \text{ decades} \]

7. \[ \frac{\text{cents}}{\text{oranges}} = \frac{6 \text{ oranges}}{3 \text{ oranges}} \times \frac{1.5 \text{ lb}}{20 \text{ lb}} \times \frac{20 \text{ lb}}{3 \text{ lb}} = 6 \times \frac{1.5}{3} \times 20 \text{ cents} \]

8. \[ \frac{\text{hr}}{\text{mi}} = \frac{405 \text{ mi}}{52 \text{ mi}} = 405 \times \frac{1}{52} \text{ hr} \]

9. \[ \frac{\text{dollars}}{\text{mi}} = \frac{405 \text{ mi}}{18 \text{ mi}} \times \frac{\frac{1.34 \text{ dollars}}{\text{mi}}}{\frac{1.34 \text{ dollars}}{\text{mi}}} = 405 \times \frac{1}{18} \times 1.34 \text{ dollars} \]

10. The question asked is "How many oranges are there in 1 crate?"

\[ \frac{\text{oranges}}{\text{crate}} = \frac{1 \text{ crate}}{1 \text{ crate}} \times \frac{\frac{1.60 \text{ dollars}}{\text{crate}}}{\frac{0.30 \text{ dollars}}{\text{lb}}} \times \frac{3 \text{ oranges}}{\frac{1}{\text{lb}}} \]

\[ = 1 \times 1.60 \times \frac{1}{0.30} \times 3 \text{ oranges} \]

11. If you are having trouble, start by writing all the conversion factors given in the problem. Then you can simply select the one you need from the collection.

\[ \frac{\text{dollars}}{\text{lb}} = \frac{1 \text{ lb}}{1 \text{ lb}} \times \frac{16 \text{ oz}}{1 \text{ lb}} \times \frac{1 \text{ sheet}}{500 \text{ oz}} \times \frac{\text{ream}}{500 \text{ sheets}} \times \frac{\$4.00}{\text{ream}} \]

\[ = 1 \times 16 \times \frac{1}{500} \times \frac{1}{500} \times 4.00 \text{ dollars} \]

12. \[ \frac{\text{cars}}{\text{min}} = \frac{1 \text{ train}}{1 \text{ train}} \times \frac{2 \text{ min}}{\text{train}} \times \frac{\text{hr}}{60 \text{ min}} \times \frac{40 \text{ min}}{\text{hr}} \times \frac{5280 \text{ ft}}{\text{mi}} \times \frac{\text{car}}{50 \text{ ft}} \]

\[ = 1 \times 2 \times \frac{1}{60} \times 40 \times 5280 \times \frac{1}{50} \text{ cars} \]

13. \[ \frac{\text{ounces}}{\text{lb}} = \frac{1 \text{ egg}}{12 \text{ eggs}} \times \frac{1 \text{ doz}}{27 \text{ doz}} \times \frac{18 \text{ lb}}{1 \text{ lb}} \]

\[ = 1 \times \frac{1}{12} \times \frac{18}{27} \times 18 \text{ oz} \]